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G. Alfred Teasley

A FURTHER ANALYSIS OF THE
FULL FLOATING TEXTILE SPINDLE BEARING

A THESIS

Presented to
the Faculty of the Graduate Division
Georgia Institute of Technology

In Partial Fulfillment
of the Requirements for the Degree
Master of Science in Mechanical Engineering

By
G. Alfred Teasley
September 1955

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A FURTHER ANALYSIS OF THE

FULL FLOATING TEXTILE SPINDLE BEARING

Approved:

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Date Approved by Chairman: December 17, 1955

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SUMMARY

Continuing the analysis of the plain bolster textile spindle, it is the purpose of this research to investigate the effect of thread load magnitude and position upon bearing friction and to analyze and experimentally investigate the oil-film caused vibration of the spindle.

The load due to thread tension can be separated into two parts: 1. a varying torque, whose speed-time effect is the useful work, and 2. a varying beam load on the spindle, which affects the load--and thus the friction loss--on both the journal and the pivot bearings. This thread caused beam load varies in magnitude, in point of application along the spindle, and in direction. In a spinning frame these changes occur at high speed and in a complex combination. In the investigation it was sought to isolate and study the first two changes listed independently. The change in direction is assumed to affect only the direction of the resultant load.

In the vibration study two methods were used in the attempt to determine the type and cause of the vibration. A visual examination of the motion of the end of the spindle using a Brinell microscope and a "Strobotac" was the first method used. In the second method an air gap in an inductance coil was placed near the end of the spindle; then the variation in the field caused by the motion of the end of the spin-

dle was recorded by use of a cathode-ray oscillograph. The natural frequency of the spindle was computed in order to have an approximation to this value for comparison with the experimental results.

It was concluded that the thread load affects the lubrication of the spindle bearings only as it affects the Sommerfeld number.

The vibration which has the greatest effect upon the lubrication of the spindle is a vibration at the natural frequency of the spindle ("oil whip"). The speed at which this vibration first occurs depends somewhat upon the viscosity of the oil used and the load on the bearings, but this speed is in every case greater than twice the natural frequency of the spindle. This oil whip generally raises the coefficient of friction above the value that would be expected at a given Sommerfeld number when using a particular oil. It also causes a variation, or "unsteadiness," in the friction produced.

The major motion of the end of the spindle is an elliptic motion at a frequency equal to the speed of rotation of the spindle. This motion is affected by the position of the belt.

It is recommended that the vibration study be continued with the purpose of discovering the effect of the bobbin and package and the thread load. Also work needs to be done concerning the thread load that can be expected in spinning.

CHAPTER I

INTRODUCTION

There are a large number of spinning spindles in operation in our textile mills today. Even though it requires only a small amount of power to drive each spindle; because of the large number of spindles in operation in an ordinary spinning room, any reduction in the amount of that power that is wasted would amount to a large saving.

Six graduate students at the Georgia Institute of Technology[1,2,3,4,5,6*] have studied various aspects of the performance of the plain cast-iron bolster type spindle. In particular Cheverton[5] and Williams[6] have isolated and studied the lubrication of the bearings of this type spindle. It is the purpose of this thesis to further that investigation with the specific objectives of 1. investigating the effect of thread load magnitude and position upon bearing friction and 2. analyzing and experimentally investigating the vibration and whirl of the spindle.

Because of the diverse nature of the two parts of the investigation, a separate chapter will be devoted to each part.

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*Numbers refer to references in Bibliography.

CHAPTER II

EFFECT OF VARYING THREAD LOAD MAGNITUDE AND POSITION UPON BEARING FRICTION

Preliminary Discussion

Thread Load Variation

The spindle in a spinning frame is loaded in a horizontal plane by the belt and the thread which is being wrapped around the bobbin and is loaded in a vertical plane by weight of the spindle and the package. Looking at the thread load more closely, it is seen that this load can be broken down into two parts: a varying torque and a varying beam load on the spindle. The speed-time effect of the torque is the useful work done by the spindle. The beam loading on the spindle, along with the belt load and the package load, determines the load on the bearings supporting the spindle. The loads on the bearings have an effect on the energy consumed in the bearings; i.e., the energy which is wasted. One of the purposes of this research is to investigate the effect of the thread load on the bearing friction.

The beam load effect of the thread load varies in magnitude, point of application, and direction. The magnitude of the thread load is dependent upon the direction and speed of travel of the rail and the various factors which influence the speed of the follower. The point of application depends upon the position of the rail. The direction changes as the follower moves rapidly around the ring. It is obvious to anyone who has watched a spinning frame in operation that the changes in beam load effect mentioned above occur rapidly and in a complex combination; therefore it is necessary to isolate the changes and study each independently.

Since the principal effect of the change in direction is a change in direction of the resultant load, no attempt is made to study this change.

No attempt is made to study the dynamic effects of the various changes in loading.

Operating Conditions for Testing

Operating conditions will be selected where previous experimental results would indicate the greatest conformity to theoretical results and the least number of complicating factors. It has been found that when operating the bearing with a Sommerfeld number in the range between ten and one hundred the results are much better behaved than when operating outside that range.[6] Therefore, the various operating factors will be chosen to give a range of points whose Sommerfeld numbers largely coincide with this range.

Since previous experimenters have had difficulty with vibration, an attempt will be made to choose operating conditions which will eliminate this distrubance. From theoretical considerations, previous work on the spindle, and the results of other experimenters; it is found that the spindle should be run with the lighter oils, with moderate unit loading, and at low speeds (see Chapter III).

Maximum Thread Load

Since no information on the thread loads encountered in spinning could be found, an attempt is made at finding an upper bound on this load in the following manner. It can be safely assumed that the tensile strength of the thread is higher than the loads encountered in spinning. Therefore if an upper bound can be found for the minimum tensile strength of the thread, then this value will be higher than--or on the "safe side"--the maximum spinning load. From a table calculated using Shendon's Formula for carded cotton yarn, the largest skein breaking strength is 254.0 pounds per skein[7] or 1.6 pounds per end. Since a skein break test is of "the weakest of a series" type, this value will be satisfactory to use as an upper bound for the thread load on a spindle.

Instrumentation and Equipment

The basic equipment was built by Cheverton and was modified by Williams. Basically it consists of the spindle, bolster, oil reservoir assembly; a torque measuring apparatus; a loading device; mounting and operating equipment; and other instrumentation. For the purpose of obtaining data on the effect of the thread load, such equipment was added as mentioned in the section on thread loading apparatus.

Spindle-Bolster-Oil Reservoir Assembly

The details of the spindle-bolster-oil reservoir assembly are shown in Figs. 1, 2, 3, and 8.

Friction Measuring Apparatus

The friction measuring apparatus consists of a constant strength cantilever beam on which two SR-4 strain gages have been cemented (see Fig. 2). The cantilever beam is loaded by attaching it to the bottom of the oil reservoir with a thread in such a way as to restrain the ball bearing mounted oil reservoir's turning with the spindle (see Fig. 1). Thus the friction torque developed in the bearing is transmitted to the cantilever beam as a point load. Since friction torque on the journal is about one percent of the friction torque on the bearing,[6] the friction torque as measured will be assumed to be the friction on the journal. The strain produced in the beam is measured and recorded on a strain-time recorder (see Fig. 6) which has been calibrated so that the friction force may be determined.

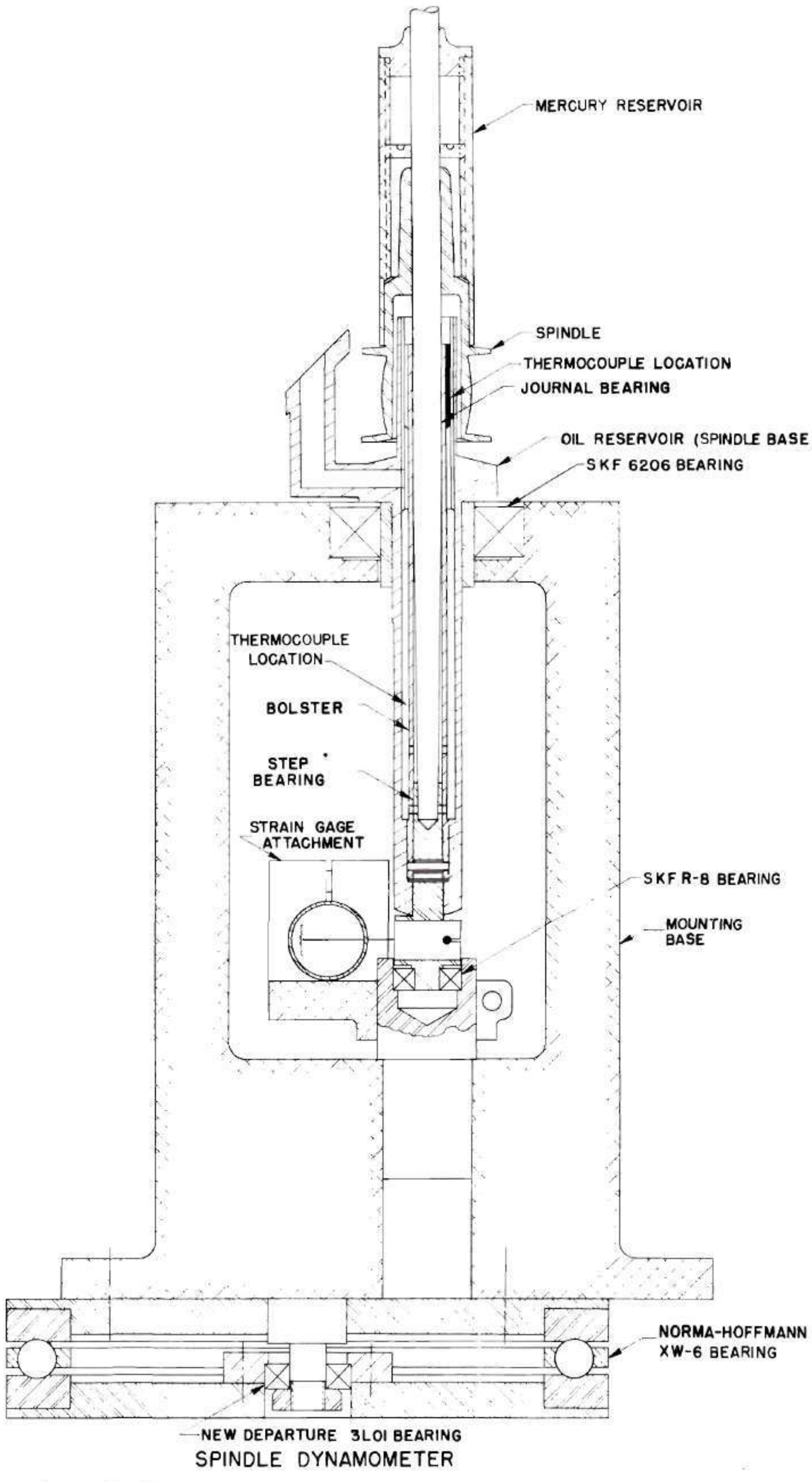

Fig. 1. Spindle Dynamometer, Assembly Section

Figure 2

Spindle Dynamometer

Component Parts of the Spindle and the Friction Force Pickup

1. Friction Force Pickup Housing and Mounting Bracket
2. Friction Force Pickup
3. Spindle with Mercury Reservoir (Mercury Reservoir Not Used)
4. Cast-iron Bolster with Journal Bearing Thermocouple Attached
5. Special Spindle Oil Reservoir

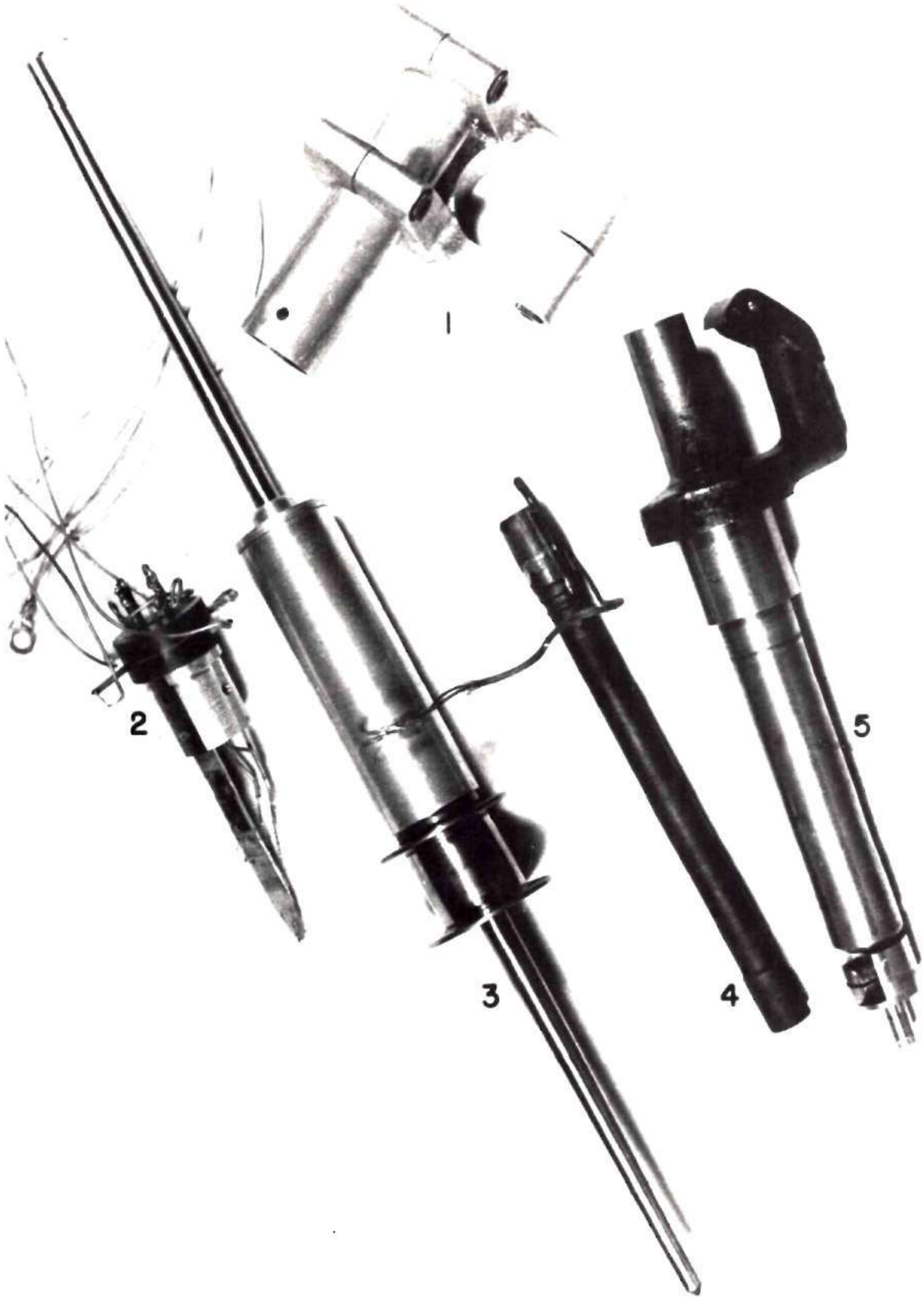

Fig. 2. Spindle Dynamometer, Component Parts of the Spindle and the Friction Force Pickup

Figure 3

Spindle Dynamometer

Assembly

1. Mounting Base
2. Mercury Slip Ring
3. Thermocouple Leads, Oil Reservoir
4. Thermocouple Leads, Journal Bearing
5. Cord of Loading Device
6. Friction Force Pickup
7. Pulley Used in Calibrating Friction Measuring Apparatus
8. Thrust Bearing Assembly
9. Weights for Calibrating Friction Measuring Apparatus
10. Weights for Producing Vertical Load (Not Used)

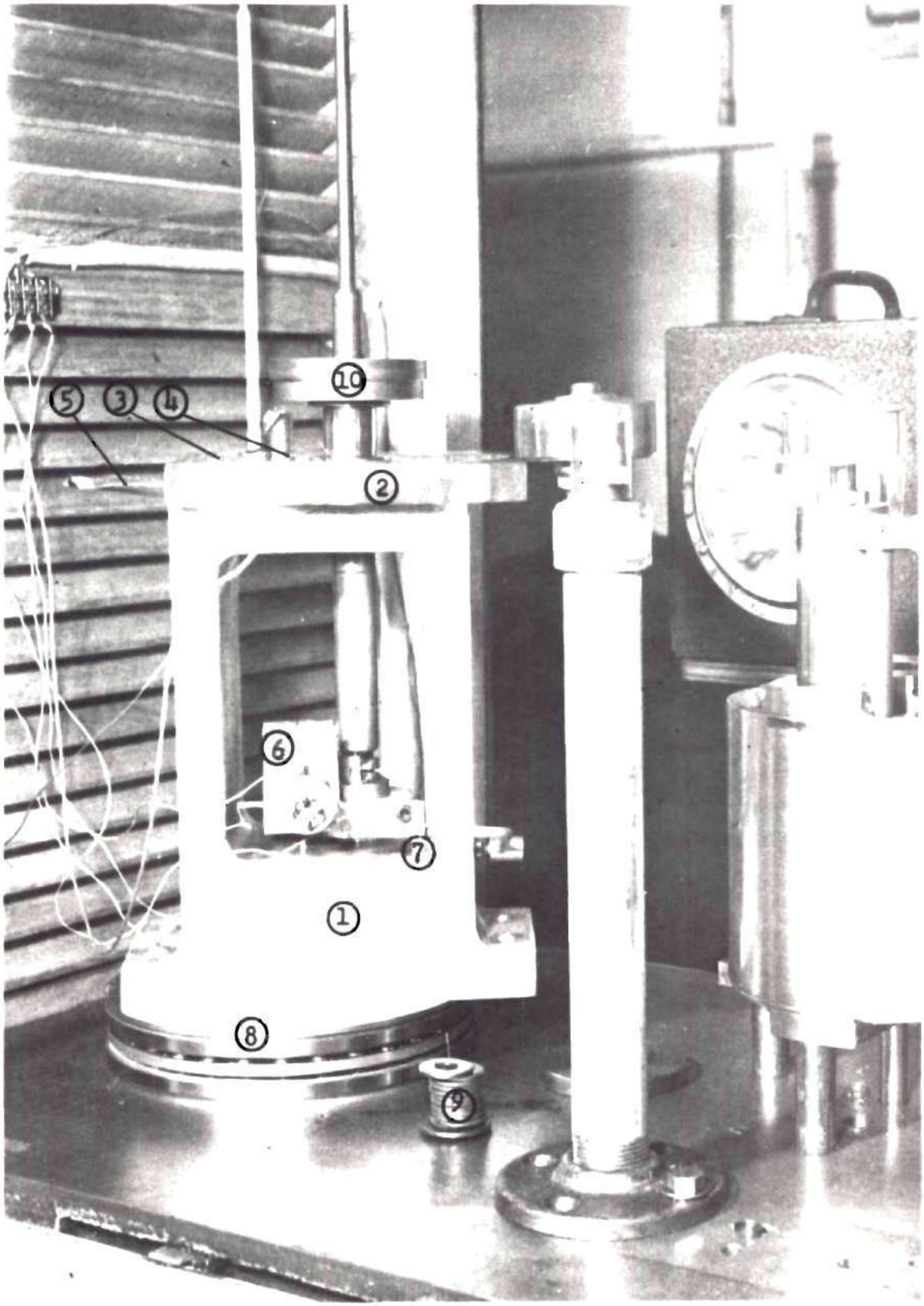

Fig. 3. Spindle Dynamometer, Assembly

Loading Device

A cord attached to the mounting base (see Fig. 5) and passing over a pulley is used to provide a known load on the bearing (see Fig. 4). If the cord load and the belt load are in the same vertical plane (the spindle is also in this plane) then the belt load is 0.947 times the cord load and the load on the bearing is 0.985 times the cord load (see Appendix, p. 63) provided there is no thread load acting on the spindle.

To insure that the cord load and the belt load are in the same vertical plane, the mounting base is mounted eccentrically on a ball thrust bearing arrangement (see Fig. 1). Thus moving the thrust bearing in a straight line by means of the adjusting mechanism (see Figs. 4 and 5), it is possible to align the loads. With the known loads aligned, there can be no loads on the bearing in any other plane since the thrust bearing mounting prevents any torques from being developed (in a horizontal plane). A sighting box (see Fig. 5) is used as an aid in this maneuver.

Figure 4

Spindle Dynamometer

Operating Equipment

1. Voltage Regulator
2. Ammeter in Driving Motor Circuit
3. By-pass Switch to Ammeter
4. Loading Pan of Loading Device
5. Synchronous Motor
6. Mechanism for Adjusting Thrust Bearing

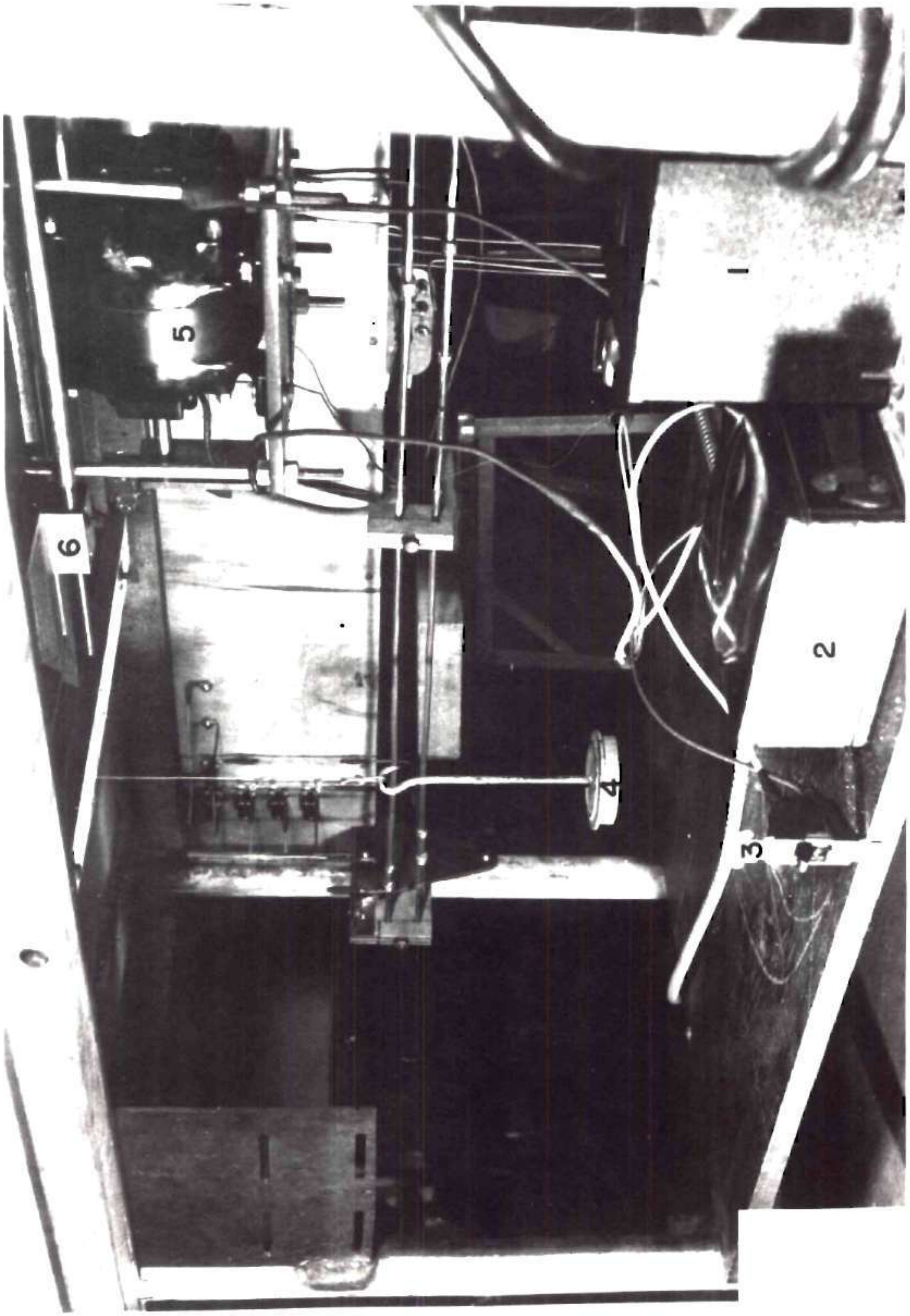

Fig. 4. Spindle Dynamometer, Operating Equipment

Figure 5

Spindle Dynamometer

Thread Loading Apparatus

1. Ball Bearing Contact Piece
2. Set Collars
3. Pulley
4. Pan
5. Weights for Thread Loading
6. Adjusting Mechanism for Pulley (Shown in Position "C")
7. Cord Loading Device
8. Mechanism for Adjusting Thrust Bearing
9. Sighting Box
10. Nine Speed Tranmission
11. Steel Mounting Plate
12. "Strobotac"

Fig. 5. Spindle Dynamometer, Thread Loading Apparatus

Figure 6

Spindle Dynamometer

Controls and Recording Equipment

1. Strain-Time Recorder
2. Portable Indicator (Used with Thermocouples, Reads Temperature Directly)
3. Rheostat and Switches for Controlling Ambient Temperature
4. "Strobotac"
5. Heating Element and Fan Housing
6. Regulated Voltage Outlet
7. Thermocouple Selector Switch

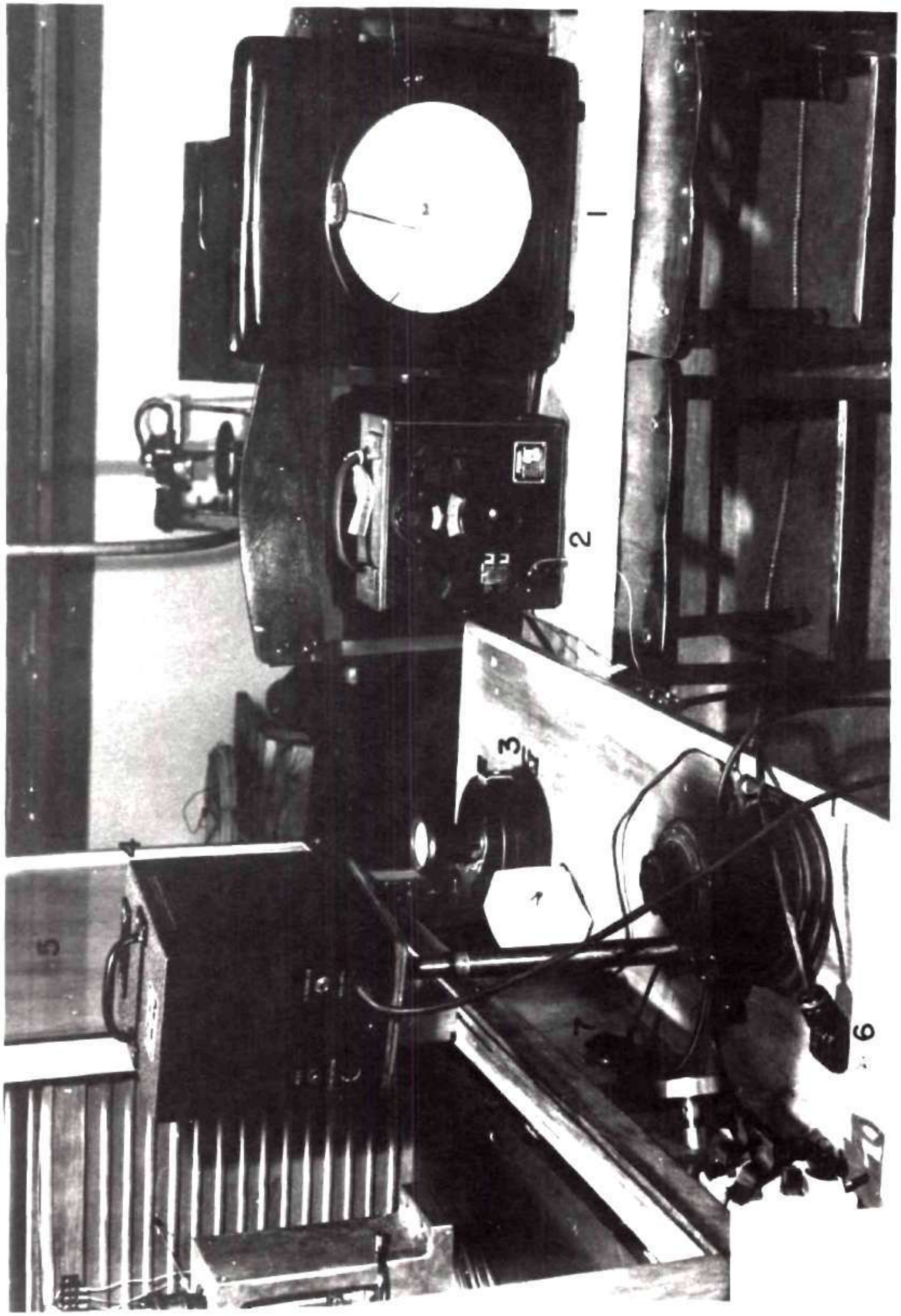

Fig. 6. Spindle Dynamometer, Controls and Recording Equipment

Mounting and Operating Equipment

The spindle is run by a synchronous motor through a nine speed transmission, allowing spindle speeds of from approximately 4000 to 12,000 revolutions per minute (see Figs. 4 and 5).

The motor, transmission, and mounting base are mounted on a heavy steel plate to prevent transmission of the vibration from the operating equipment to the spindle (see Fig. 5). The spindle is enclosed in a glass and wood compartment in which it is possible to have some control over the dry bulb temperature. A number of light bulbs and a circulating fan provide the controlling elements.

Other Instrumentation

The oil temperature is measured by means of two thermocouples; one of which is cemented to the back of the journal (see Fig. 2) and the other hangs loose in the oil reservoir (see Fig. 1). Since the temperature drop through the bolster is less than 1°F,[5] the temperature as read will be considered the oil film temperature. A mercury slip ring (see Fig. 3) makes it possible to take torque readings and temperature readings simultaneously. A temperature indicator is used so that the temperature can be read directly (see Fig. 6).

A "Strobotac" is used to determine the speed of the spindle (see Fig. 6).

The "Strobotac" and the strain recording instrument are fed through a voltage regulator (see Fig. 4) for more accurate measurements.

Thread Loading Apparatus

The apparatus added to simulate the thread load consists of a ball bearing contact piece with the thread attached thereto, a group of four set collars, a pulley which can be mounted in four different positions, and a pan and weights (see Fig. 5).

Experimental Procedure

General

Care of ball bearings.--Since static friction in the ball bearings would tend to develop torques of undeterminable magnitudes, it is important that all the ball bearings are kept clean and oiled with only enough very light oil to prevent rusting. This care will reduce any static friction to a minimum.

Calibration of the friction measuring apparatus.--Before the friction measuring apparatus can be used it must be calibrated to read pounds of force. This calibration is done by extending the thread around the base of the oil reservoir and over a pulley and then loading the thread with weights of known magnitude. Several runs are made by progressively loading the beam, allowing enough time at each level for the beam to reach its full deflection. A graph of loading weight, or force on the beam, versus chart reading is then plotted (see Fig. 17).

While the calibration is being done it is important to have the motor running (with the drive belt off) to produce vibration to help reduce the effect of static friction in the ball bearings. However, due to the sturdy construction of the mounting components, there is some noticeable effect of the static friction, particularly at very low loads. Because, when the spindle is running, considerably more vibration is transmitted to the ball bearings, it is believed that the greatest error in a reading would occur during calibration. For this reason it is believed that the greatest error in the friction force reading is $\pm$ 8.8 grams in the range where most of the readings occur.

Position of bolster.--It was found that the position of the bolster has a considerable effect upon the friction developed.[6] To prevent adding another variable, a position for the oil reservoir--and thus for the bolster--was selected. This position is such that the line of action of the cord load passes over the right-hand shoulder of the filling tube. In this way the bolster can be positioned within 5° of the normal position.

The reason for positioning the bolster in the given location is to consistently position the thermocouple which measures the temperature of the oil in the journal as close to the point of minimum film thickness as possible.

Determination of viscosity.--The viscosity of the oil is read from Fig. 16 which was drawn from data supplied by the Texas Company (see Appendix, p. 57).

Ambient temperature.--Since the data for this thesis was taken during warm weather, the heat generated by the bearings and transmission was enough to keep the ambient temperature up to at least the selected temperature of $85^{\circ}F$. The circulating fan was run to prevent the buildup of the temperature around the oil reservoir. At times the glass sides and top had to be arranged so as to draw in enough cooler air from the room to keep the temperature down to the prescribed level.

Determination of speed.--Because a belt drive does not give an exact speed ratio it is necessary to determine the speed of the spindle by the use of a "Strobotac."

Attachment of the thread load.--When positioning the ball bearing contact piece it is important to use the two set collars which are closest to the diameter of the shaft at that point, center the set collars carefully to prevent vibration, and allow a slight amount of play along the shaft for the bearing.

Preliminary

At the start of each day's runs it is necessary to warm up the strain recorder and the "Strobotac." While this warm up is proceeding the spindle is started and run under the first operating conditions (oil, speed, load, etc.) to be investigated. The correct ambient temperature is also obtained during this time. After the instruments mentioned have stabilized, they and the temperature recorder are zeroed. Then the runs are ready to be made.

Step by Step Procedure

The steps in making a run are as follows:

1. The bearing is loaded with the cord load.
2. The desired thread load is applied.
3. The correct line of action of the cord load and the thread load is obtained.
4. The oil reservoir is positioned.
5. Sufficient time is allowed for thermal equilibrium to be established.
6. The chart drive on the strain recorder is turned on.
7. The run is made for sufficient length of time. Normally the runs were for four minutes; however for the runs where the friction torque was not steady a longer run was necessary.
8. The chart drive is turned off.

Test Runs

Test runs are made at three spindle speeds, with two oils, with several cord loads, with the thread load at four positions along the spindle, and with five values of thread load varying from 0.0 pounds to 1.6 pounds. Four series of runs are made with the approximate speed and the oil used grouped as follows:

I. spindle speed 4000 rpm, oil E.

II. spindle speed 6300 rpm, oil A.

III. spindle speed 4000 rpm, oil A.

IV. spindle speed 4800 rpm, oil E.

The runs with no thread load are used as "reference" runs.

Since it was found that the temperature stabilized more quickly when it rose than when it descended, runs are made in the order of ascending oil film temperature as far as it is possible.

Discussion of Results

The interpretation of the data obtained in tests of the spindle bearing is made difficult by the fact that it is impossible to separate the effect of the journal bearing from the effect of the pivot bearing. Also adding to the complication is the fact that during the runs with thread loading an appreciable radial load is placed on the pivot bearing. Any method of interpretation selected will therefore be, at least to some extent, arbitrary.

The method chosen consists of finding a force to be used as the load on the journal bearing by adding (disregarding any difference in direction) the resultant force on the journal bearing (taking into account the effect of both the thread load and the belt load) to the resultant force on the pivot bearing (see Appendix, p. 69). In other words, it is assumed that the load on the journal bearing is the "total radial load."

The results of the runs with thread load are tabulated in Table 5 and are plotted in Fig. 7. The theoretical curve and the coordinate system (which is proportional to the inverse hyperbolic sine) are taken from an article on the analysis and design of journal bearings by John Boyd and Albert A. Raimondi.[8]

From Fig. 7 it is seen that the experimental points generally fall in a band which parallels the theoretical curve, except at low Sommerfeld numbers. A comparison of the points which represent operating conditions with thread load with the "reference" points and with the results obtained by Williams (his Fig. 8)[6] shows that the thread load causes no marked effect upon the distribution of the points. Therefore it is shown that the thread load magnitude and position affect the friction only as it changes the Sommerfeld number.

It will be noticed in Table 5 that for some runs more than one chart reading (and thus the coefficient of friction variable) and/or value of temperature (and thus the Sommerfeld number) are recorded. This "double point" is recorded because of one of two phenomena. First, during the run the chart reading and/or the temperature would remain more or less steady at one value; and then, for no apparent reason, the value of this reading would change--sometimes suddenly, sometimes over an interval of time--and would remain at some new

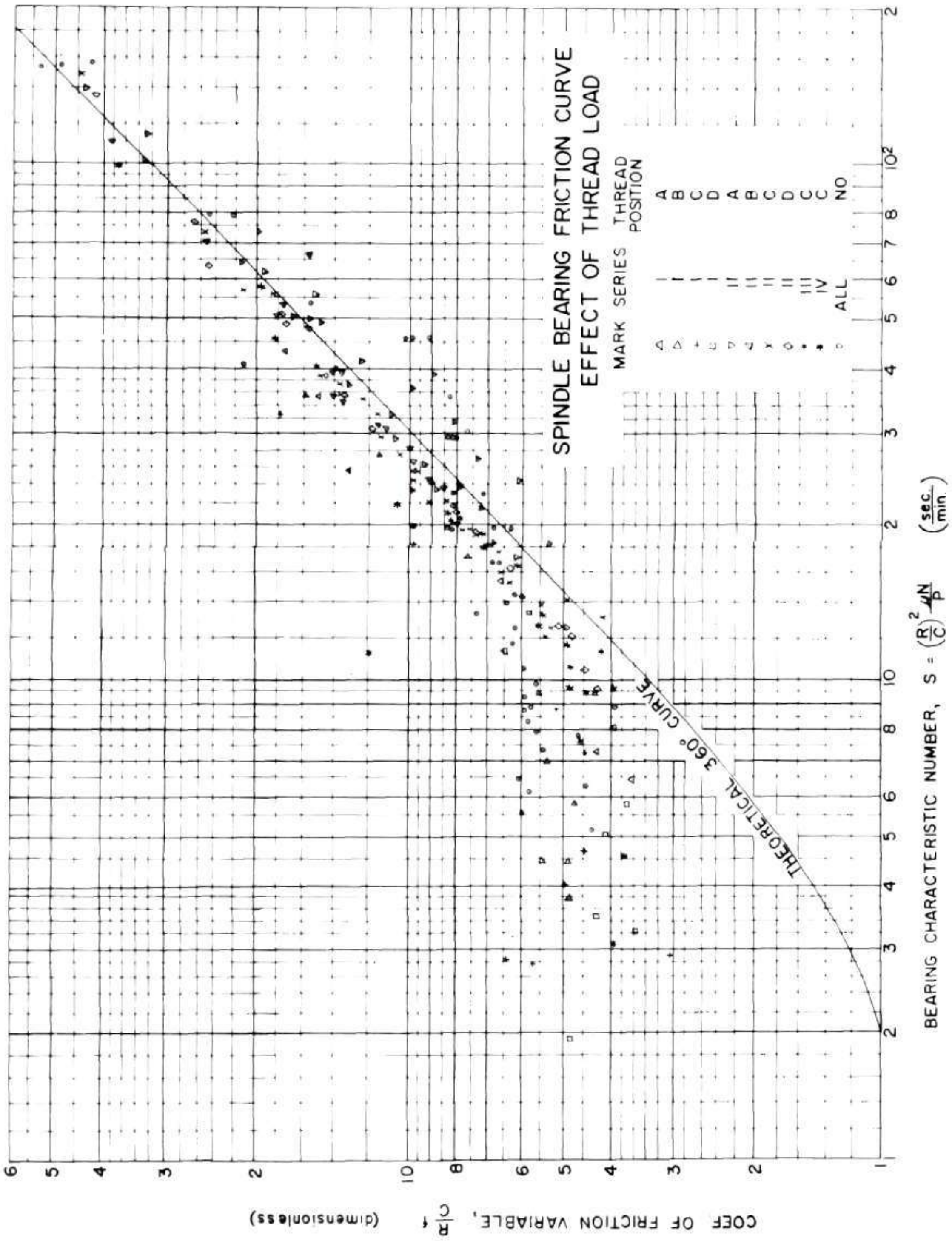

Fig. 7. Spindle Bearing Friction Curve, Effect of Thread Load

value. Second, in a few cases the value of the chart reading would fluctuate between two values and would show a tendency to remain at these extremes.

Some variation of the friction reading occurred spasmodically during the taking of data. It tended to occur for several consecutive runs and at low belt loads. It is believed that the reason for this variation was excessive belt slippage. During the vibration tests it is known that excessive slippage occurred once at a low belt load, resulting in a chart reading similar to that mentioned above. After the belt was wiped dry of as much oil as would wipe off, the chart variation stopped. The fact that the drive belt was considerably worn probably aided the slippage.

Visual inspection of the pivot bearing showed continued polishing due to metal to metal contact between the spindle and the bolster. Also evident after the runs at the lower Sommerfeld numbers was a small amount of polishing at the bottom of the journal bearing.

CHAPTER III

ANALYSIS AND EXPERIMENTAL INVESTIGATION OF THE VIBRATION AND WHIRL OF THE SPINDLE

Preliminary Discussion

General

With the advent of higher speeds of operation of journal bearings, the problem of spindle vibration and oil-film whirl became of considerable importance. Because of the complexity of the problems which arise in connection with vibration and whirl, during the short span of years in which these problems have been of recognized importance no comprehensive study, either theoretical or experimental, has been assembled. Since no standardized terminology has been established in this field, the terms will be defined as they are discussed.

Oil-film Whirl

Oil-film whirl is a self-excited vibration in which the journal center translates in a closed path in the direction of journal rotation. The frequency of this vibration has been determined to be one-half the running speed or less, the exact value depending upon such factors as the mean attitude of the path and the mass of the journal.[9]

When unloaded, a plain journal bearing will whirl at all speeds of operation.[10] However, for a loaded bearing there is a minimum speed below which oil-film whirl will not occur; it would seem that the mechanics of the determination of this speed are not completely understood at this time.

That the possibility of the occurrence of oil-film whirl is contained in the general hydrodynamic equations is shown by Shaw and Macks.[9] Poritsky obtains a solution of this type for small displacements and very low loads.[10]

Three methods have been found in the literature to reduce the tendency for oil-film whirl. Briefly these methods are 1. design the bearing to operate under appreciable unit load by limiting the bearing area,[9] 2. groove the bearing in such a way as to increase unit pressure,[9] and 3. groove the journal (or the bearing) to change the slope of the eccentricity locus at the origin.[11]

Oil Whip

"Oil whip of a rotor consists in a whipping or whirling motion of the rotor shaft, of frequency essentially equal to the critical frequency of the rotor; it occurs at rotor speeds roughly exceeding double the critical speed."[10]

Poritsky explains the mechanics of the oil-film forces which are responsible for oil whip.[10] [a] It is possible that oil whip is caused by, or at least abetted by, the hysteresis

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[a] It is seen from the article cited that oil-film whirl occurs when the radial componènt of the force is negligible; but when this component is not negligible it stabilizes the oil-film whirl, but it does not stabilize--and in fact explains--oil whip.

of the shaft metal.[9]

Newkirk and Lewis report that the "whirl-impending" (whip-impending) speed varies from two to six times the critical speed. The factors which affect the value of the "whirl-impending" speed are the eccentricity ratio, the L/D ratio, the unit load, and the clearance ratio.[12]

In general, the more stable bearings seem to be those which operate at high eccentricity ratios, at moderate unit loadings, with lighter oil, and with larger clearance ratios. It was also found that slight misalignment of the bearing results in a more stable condition.[12]

Method of Attack

The usual method to study the effect of oil-film forces is to observe the end of a shaft which stops just beyond the outboard end of the bearing. Since this method is impossible when studying the spindle, the end of the spindle will be used for a reference. Because of the use of this method, the problem must be carefully analyzed in order to eliminate possible extraneous (extraneous to the problem of oil-film forced vibration) occurrences.

Theoretical Determination of the Critical Speed of the Spindle

Assumptions

In determining the natural frequency of the spindle it is assumed that the spindle is a simply-supported shaft with an overhanging end. It is also assumed that the supports are rigid. Because of this last assumption--that the "flexibility" of the oil film, bolster, and oil reservoir is zero--the calculations would indicate a higher natural frequency than is actually present. Since the "flexibility" of the oil film is not necessarily constant for all speeds, the actual natural frequency may also vary.[10] The effect of gyroscopic action is neglected.

Method

The method to be used is the method developed by M. A. Prohl.[13] However this method must be modified to take into account the effect of the overhanging portion of the spindle. The theory necessary is developed in the Appendix.[b]

A drawing of the spindle as obtained by field measurements is shown in Fig. 8 and the equivalent system is shown in Fig. 9 (the values for the equivalent system are given in Table 6).

Results

Figure 10 gives the plot of excess bending moment versus assumed speed. From the figure it is seen that the

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[b]It should be noted that the method used in developing the necessary additional theory is not the same as used by Prohl, but it is another method that can be used to obtain the same results and is by W. T. Thompson.[14]

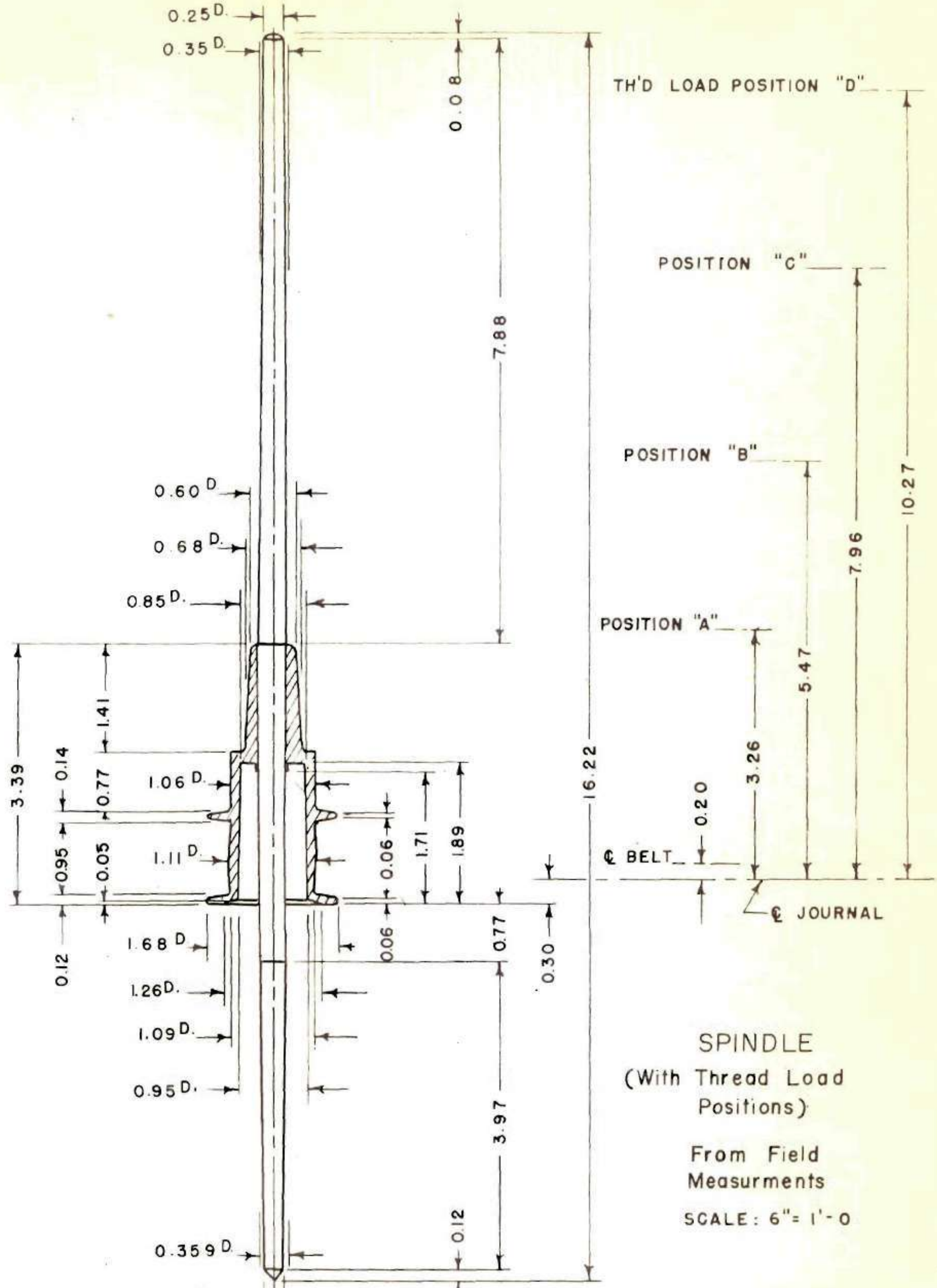


FIG. 8. SPINDLE DRAWING

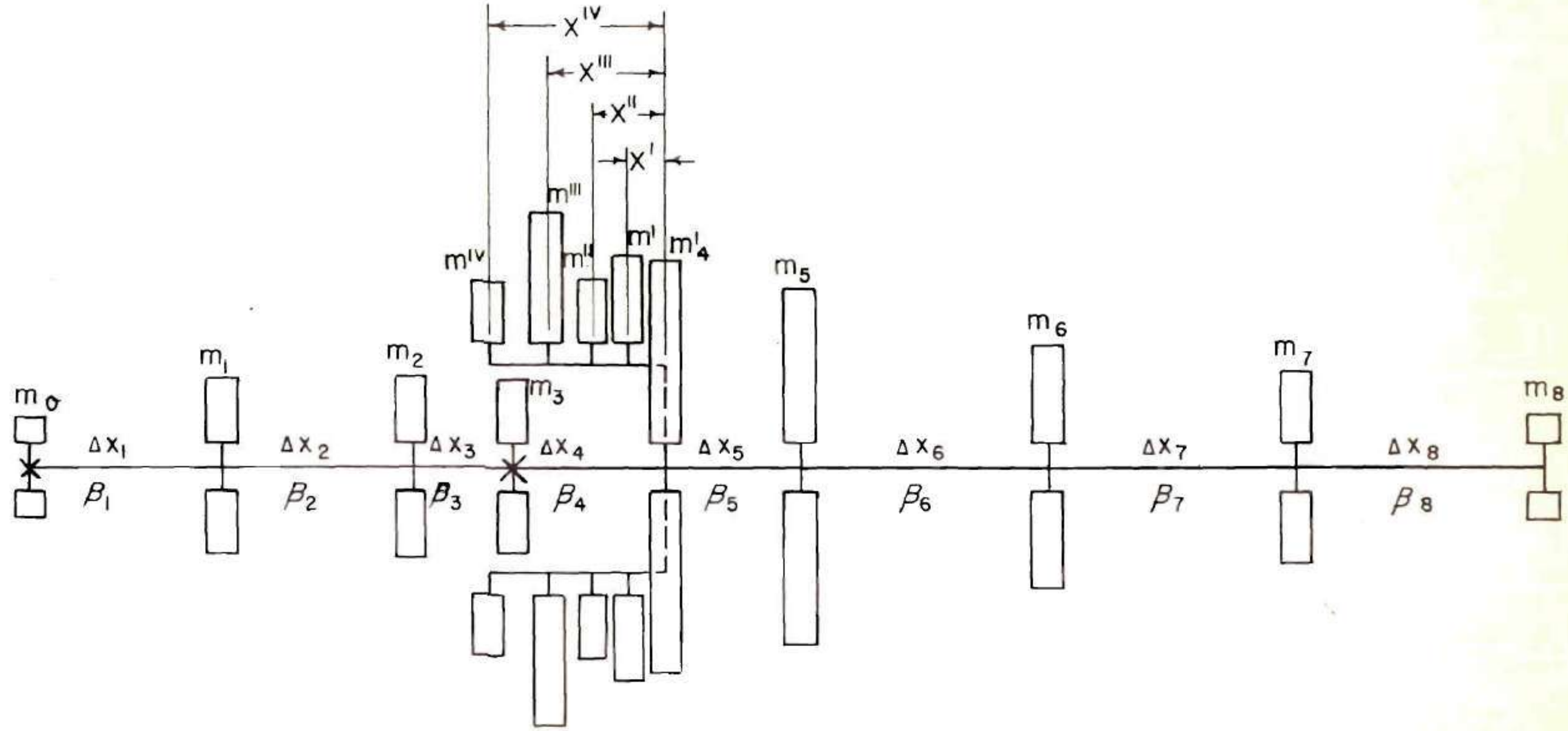


FIG. 9. SPINDLE, EQUIVALENT SYSTEM

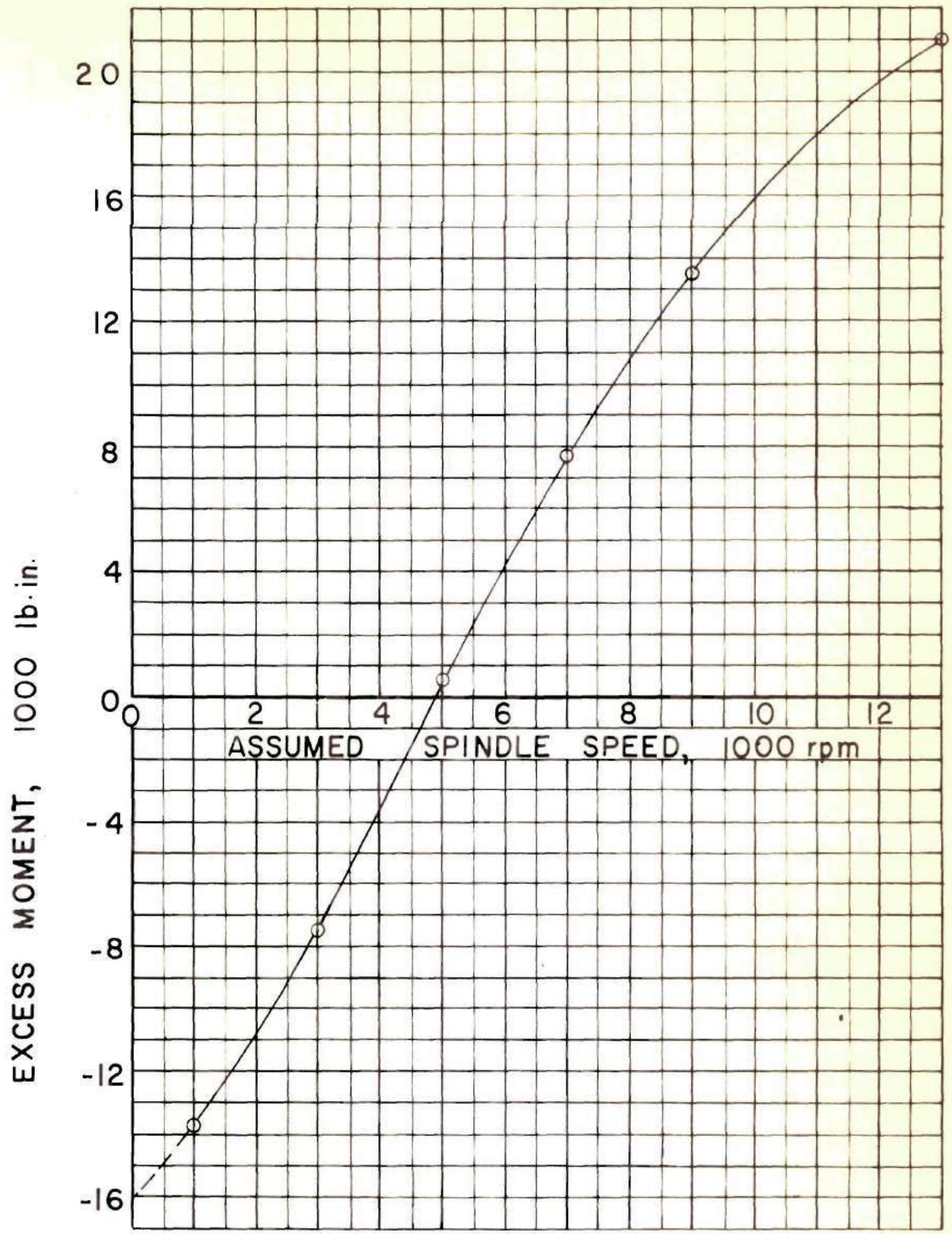


FIG. 10. SPINDLE, DETERMINATION OF NATURAL FREQUENCY, EXCESS MOMENT VS ASSUMED SPINDLE SPEED

first natural frequency of the spindle is approximately 4900 cycles per minute and that the second natural frequency is well beyond the upper limit of speed of operation.

Instrumentation and Equipment

General

In general the instrumentation is the same as that described in Chapter II in the sections on spindle-bolster-oil reservoir assembly, friction measuring apparatus, loading device, mounting and operating equipment, and other instrumentation. In addition, two distinct methods used to obtain information on the vibration are described below.

Visual Method

For visual observation of the motion of the end of the spindle a Brinell microscope is mounted so as to have the end of the spindle in its field of view. The "Strobotac" is mounted so that it can be used as the light source for the microscope. (See Fig. 11 for a view of the apparatus.)

The end of the spindle is modified by grinding it to the general shape of a frustum of a cone (from its original shape of a hemisphere). This modification is done in order to have a definite small-area-reflecting surface.

Figure 11

Vibration Study, Visual Method,

Equipment

1. Brinell Microscope
2. Mounting and Adjusting Bracket
3. "Strobotac"

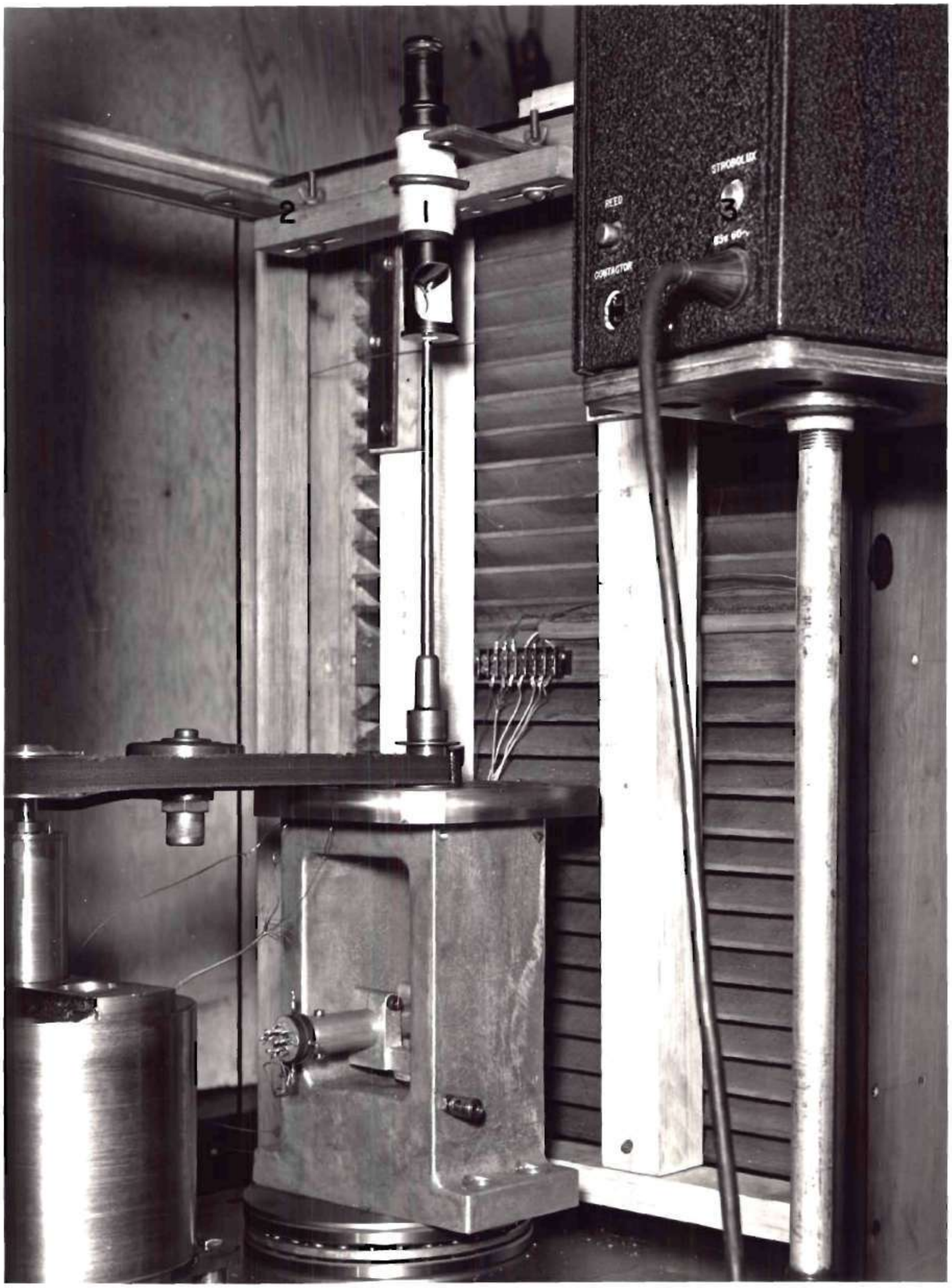


Fig. 11. Vibration Study, Visual Method, Equipment

Variable Inductance Pickup Method

A horseshoe shaped inductance coil with an air gap of 0.04 inches is mounted so that the shaft is directly opposite to and as close as possible to the air gap. The coil is mounted so that it can be easily pivoted to take readings from two positions 90° apart (the position shown in Fig. 12 is position A, position B is 90° clockwise--looking down--from position A). Three number six dry cell batteries wired in series form the power supply. A 3,300,000 ohm resistor is placed in series with the coil. A cathode-ray oscillograph is placed across the resistor. An audio oscillator is arranged so that it can provide a sweep signal of variable and known frequency to the oscillograph. (See Fig. 12 for a view of the apparatus.)

Experimental Procedure

General and Preliminary

The discussion under the two headings General and Preliminary in Chapter II is valid except for the parts dealing with thread loading.

One small difference concerns the ambient temperature. Since to clear the equipment it was necessary to keep the glass top open, the ambient temperature was not controlled as carefully during these runs as during the runs with thread loading. However it was noted during the runs that the ambient temperature stayed within two degrees of 85°F.

Figure 12

Vibration Study, Variable Inductance Pickup Method

Equipment

1. Mounting Bracket
2. Pivot
3. Inductance Coil
4. Shielded Leads
5. Dry Cell Power Supply
6. Oscillograph
7. Audio Oscillator

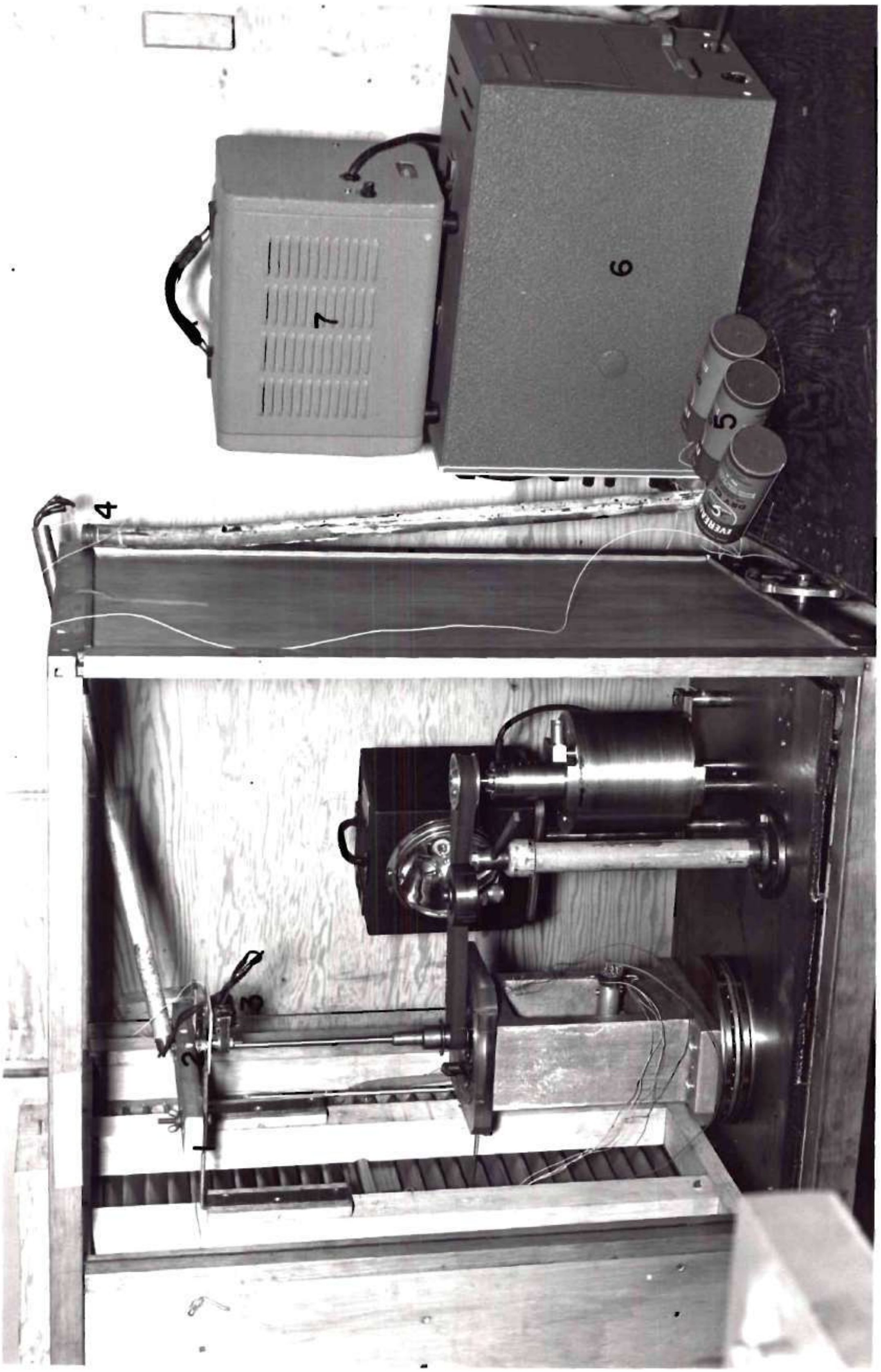


Fig. 12. Vibration Study, Variable Inductance Pickup Method, Equipment

Visual Method

Method.--The purpose of the visual method is to observe the motion of the end of the spindle.

The plan is to "stop" the motion of the end of the shaft by use of the "Strobotac," thus determining the frequency of the vibration, and then by changing the frequency of the flash of the "Strobotac" to allow the end of the shaft to move slowly in its orbit to determine the form of the vibration. In the case of a complex vibration, it is hoped that the components can be separated and studied.

Step by step procedure.--The step by step procedure to make a run is as follows:

1. The bearing is loaded with the cord load.
2. The drive motor is turned off, the correct transmission gear ratio is set, and the drive motor is turned on.
3. The correct line of action of the cord load is obtained.
4. The oil reservoir is positioned.
5. Sufficient time is allowed for thermal equilibrium to be established.
6. Visual observations are made as mentioned above.
7. The chart drive is turned on and allowed to run for two minutes (longer if necessary) during which time the value of the temperature is recorded.
8. The chart drive is turned off.

Variable Inductance Pickup Method

Method.--The purpose of the variable inductance pickup method is to determine the frequency of the vibration and to separate (if possible) the various influencing factors.

To accomplish this purpose a wave form for the vibration is obtained on the oscillograph and is studied. (The regular saw-tooth-sweep signal is used.) Then the oscillator is used to supply a sine-wave signal of such a known frequency as to obtain a single loop pattern of the oscillograph. By use of the known frequency and the relative scale lengths of the respective waves, other frequencies may be roughly determined.

Step by step procedure.--The step by step procedure for a run is as follows:

1. The bearing is loaded with the cord load.
2. The drive motor is turned off, the correct transmission gear ratio is set, and the drive motor is turned on.
3. The correct line of action of the cord load is obtained.
4. The oil reservoir is positioned.
5. Sufficient time is allowed for thermal equilibrium to be established.
6. The chart drive is turned on and allowed to run for two minutes (longer if necessary) during which time the value of the temperature is recorded.

7. During the time the strain recorder drive is on, the patterns mentioned above are studied and recorded with the pickup at two locations 90° apart.
8. The chart drive is turned off.

Test Runs

Test series for both methods were made under approximately the same operating conditions. These series were with the lightest oil, with a medium oil under several loads, and with the heaviest oil; all series consisting of runs at all possible speeds.

Discussion of Results

Non-oil-film-caused Vibration

It was found by use of the visual method (which was run first) that the vibration is very complex and perhaps somewhat erratic. It was determined, however, that one important component of the motion of the end of the spindle is at or near the speed of rotation of the spindle or is a multiple or submultiple thereof.

By use of the variable inductance pickup method it was determined that this largest component of the motion is a cyclic motion at a frequency equal to the speed of the spindle. Also present are higher frequency effects as will be noted from the wave form sketches (see Table 9). Since no reference can be found in the literature on oil-film effects to this type of "vibration," it is probable that these components do not arise from the oil-film forces. However, since these components are the largest in amplitude, it may be that they are responsible for the "wobble" of the spindle.

The amplitude of this "vibration" is generally greatest in the middle of the range of speed. Also it was noted that the amplitude at right angles to the belt is generally greater than the amplitude in the direction of the belt by a ratio of approximately three to one. Thus this "vibration" may be caused, and is at least affected, by some belt condition.

That this motion is sometimes very erratic, *i.e.*, it does not repeat itself exactly on every cycle, was clearly shown by the several parallel traces which were formed on the oscillograph for some runs. No definite pattern can be found for the occurrence of this phenomenon.

Since this "vibration" occurred for all runs, it is not possible to say what its effect on the friction might be. However, since this "vibration" is probably not oil-film caused, it probably causes little effect on the friction.

Oil-film Whirl

At times a "traveling wave" effect--a slow, slight, periodic (at any point), vertical vibration of one part of the trace with respect to another part--which traveled from left to right across the oscillograph screen was noted. It was noticed that when one peak went up, the adjacent peaks went down; thus showing that the frequency was slightly less than one-half the speed of the spindle (the frequency of the largest component of the motion). These facts show clearly that oil-film whirl occurred at various times.

This oil-film whirl was noticed more often in the middle of the range of speeds. However, since when the trace behaved erratically the occurrence of oil-film whirl would be hidden, a comprehensive survey of when oil-film whirl occurs in the operation of the spindle is impossible.

However, it is believed that the effect--if any--of oil-film whirl upon the friction is not significant enough to be noticed (see Fig. 13).

Oil Whip

When the spindle is operated under conditions of high speed, heavy load, and/or high viscosity a vibration was noted whose frequency was calculated to be between approximately 3000 and 4800 cycles per minute. All the speeds where this vibration was evident were above twice the natural frequency of the spindle. It is seen that this vibration meets the requirements for oil whip.

Despite the precautions taken to avoid outside interference on the oscillograph, some power line (3600 cycles per minute) disturbance was noticeable. Since the range of natural frequencies of the spindle covers this frequency, it was impossible to definitely tell when, and if, a small

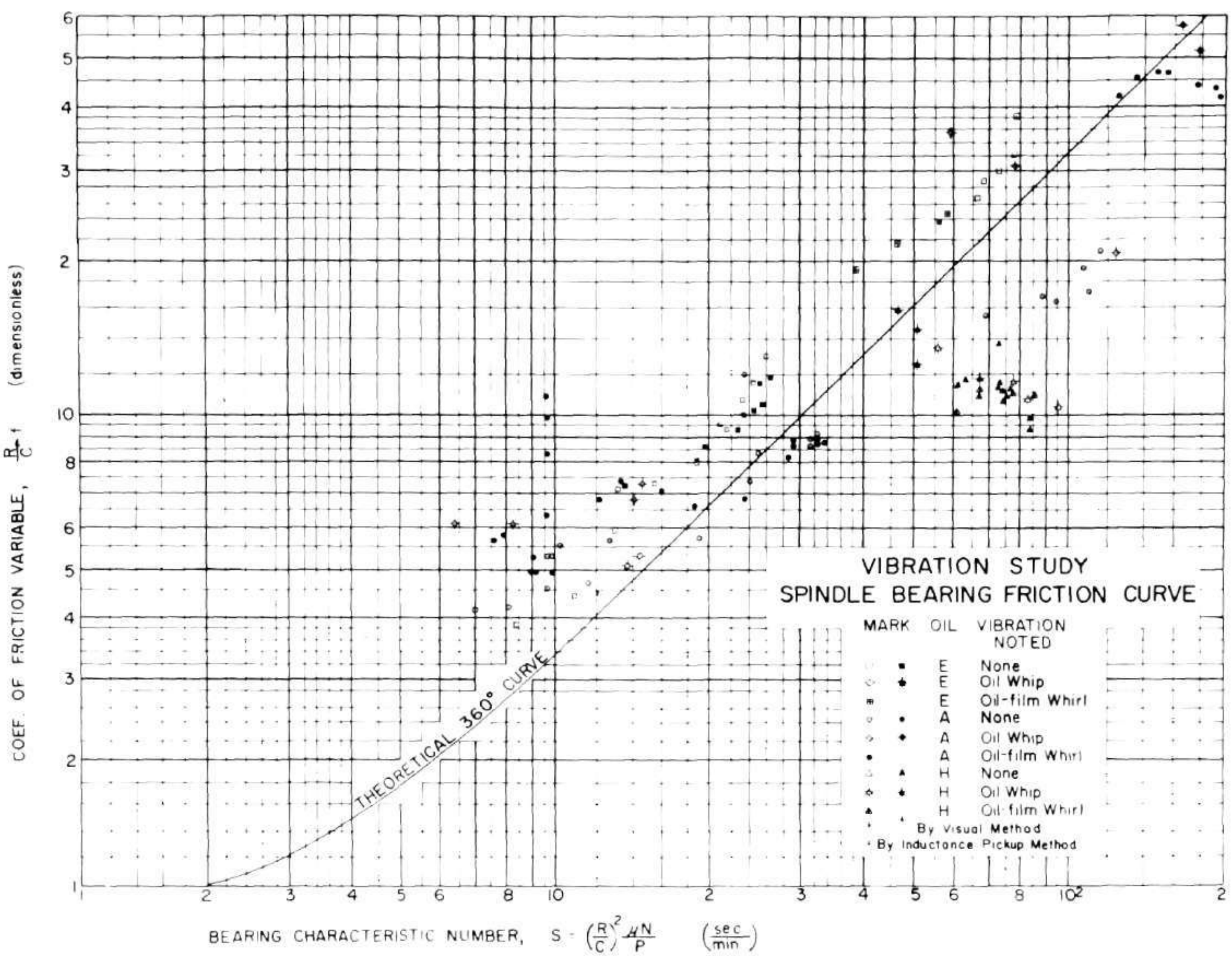


Fig. 13. Vibration Study, Spindle Bearing Friction Curve

oil whip effect had been set up. Therefore, it is possible that oil whip occurred at speeds lower than those indicated.

Based on the limited data available it is believed that the frequency of the vibration varies inversely with the speed and tends to vary with the oil used (see Table 11). That there would be some sort of variation of natural frequency with the above factors was expected, and thus this variation strengthens the belief that this vibration is due to oil whip.

During the runs with the visual method a peculiar "picture" (type IV, Table 8) was observed when the spindle was run under operating conditions similar to those under which it has been established by use of the variable inductance pickup method that oil whip does occur. Therefore it is logical that this type of "picture" is a peculiar one which positively identifies oil whip. It is interesting to note that the mass of dots in the "picture" is not definitely directional in nature; while the orbit in which they travel is directional (as mentioned in the section on non-oil-film-caused vibration).

Oil whip causes a definite rise in the friction produced with a given oil (see Fig. 13). Oil whip is usually accompanied by a vibrating or unsteady line on the friction measuring chart. In Fig. 14 is shown a chart taken when the visual method was being used. (Table 1 gives pertinent data.)

Table 1. Vibration Study, Visual Method,

Pertinent Data for Fig. 14.

Run	Speed rpm	Oil	Load lbs.	Sommerfeld Number	Coef. of Friction Variable	Oil Whip Noted
1	3910	A	0.532	123.1	41.9	no
2	4880	A	0.532	133.3	45.9	no
3	5840	A	0.532	147.4	46.4	no
4	6800	A	0.532	157.8	45.9	no
5	7850	A	0.532	178.3	44.2	no
6	8840	A	0.532	194.8	41.6	no
7	9800	A	0.532	193.1	43.6	no
8	10640	A	0.532	179.2	51.9	yes
9	12000	A	0.532	163.0	57.0	yes
10	3950	A	3.035	18.5	6.61	no
11	4920	A	3.035	23.7	6.85	no
12	5880	A	3.035	28.3	7.10	no
13	6890	A	3.035	28.9	7.57	no
14	7900	A	3.035	32.2	7.68	no

This table is an excerpt from Table 8.

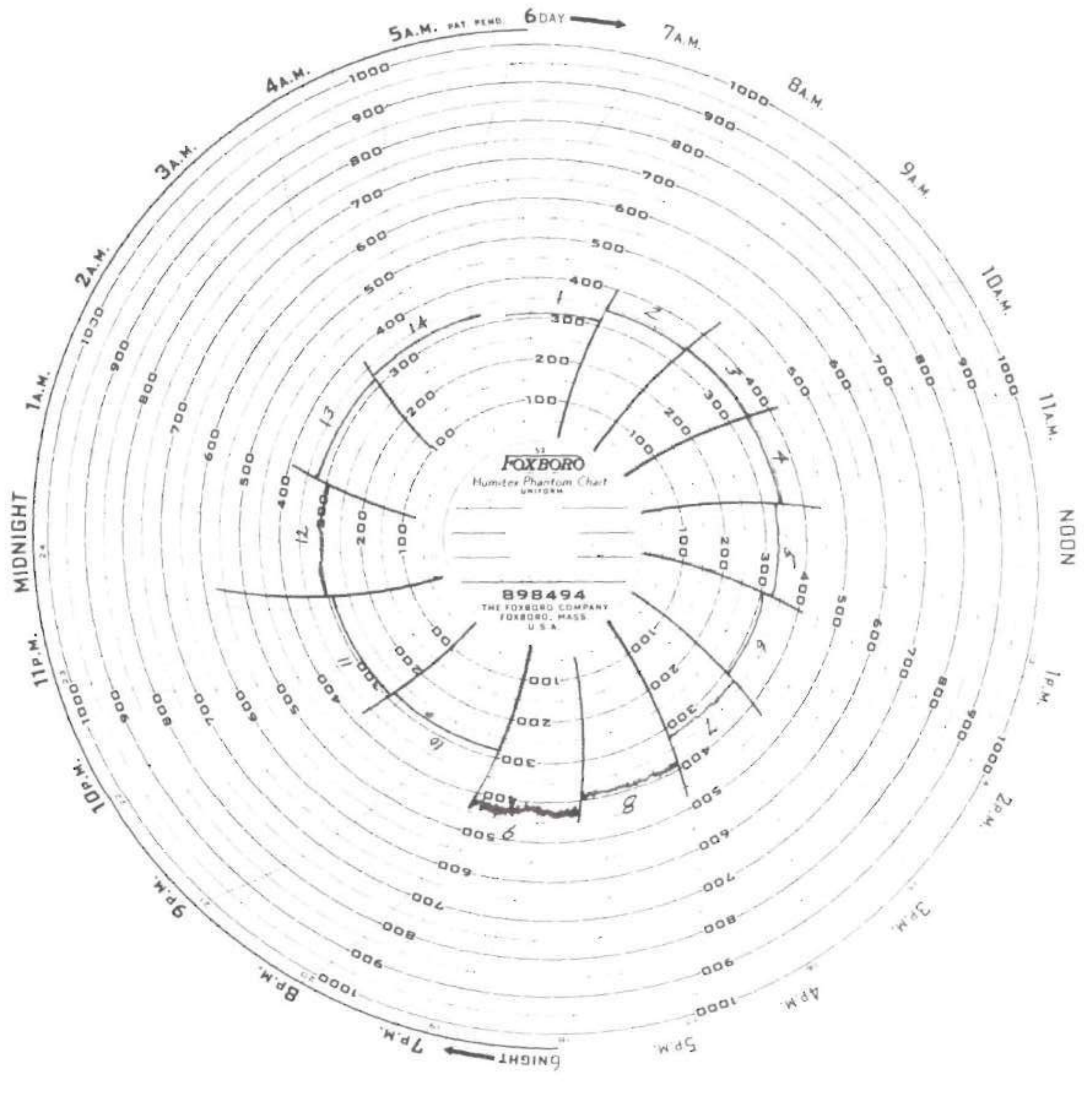


Fig. 14. Vibration Study, Visual Method, Strain-time Recording Chart

In Fig. 15 is shown a chart in which the variable inductance pickup method was being used. (Table 2 gives pertinent data.) Both of these charts are good examples of the effect of oil whip on the friction. It is not believed, however, that either type of chart-reading vibration is proof that oil whip is occuring. In fact, there is considerable evidence that other factors, such as excess belt slippage, can cause similar traces.

General Discussion

Some of the data taken in the vibration study rely upon the judgment, accuracy, and memory of the observer (examples, the oscillograph trace and the "picture" in the microscope). Due allowance was taken in the discussion of the results for the discrepancies which this dependency might cause.

It is believed that the vibration results may not be completely reproducible, especially in such particulars as the shape of the wave form on the oscillograph, the exact type of trace on the friction recording chart, and at some points where there is a tendency for the type vibration to appear and disappear without apparent cause. However, the general trend is reproducible, and thus this research is a reliable guide to the effect of the oil-film caused vibrations.

Table 2. Vibration Study, Variable Inductance Pickup Method

Pertinent Data for Fig. 15

Run	Speed rpm	Oil	Load lbs.	Sommerfeld Number	Coef. of Friction Variable	Oil Whip Noted
1	6650	H	3.035	77.0	11.2	no
2	7600	H	3.035	85.4	10.9	no
3	8560	H	3.035	83.7	10.7	yes
4	9500	H	3.035	95.3	10.1	yes
5	10450	H	3.035	77.5	11.4	yes
6	11390	H	3.035	56.5	13.4	yes
7	3940	E	0.532	38.2	19.0	no
8	4810	E	0.532	46.6	21.8	no
9	5700	E	0.532	55.2	23.9	no
10	6630	E	0.532	58.4	24.8	no
11	7570	E	0.532	66.7	26.8	no
12	8540	E	0.532	68.9	28.5	no

This table is an excerpt from Table 9.

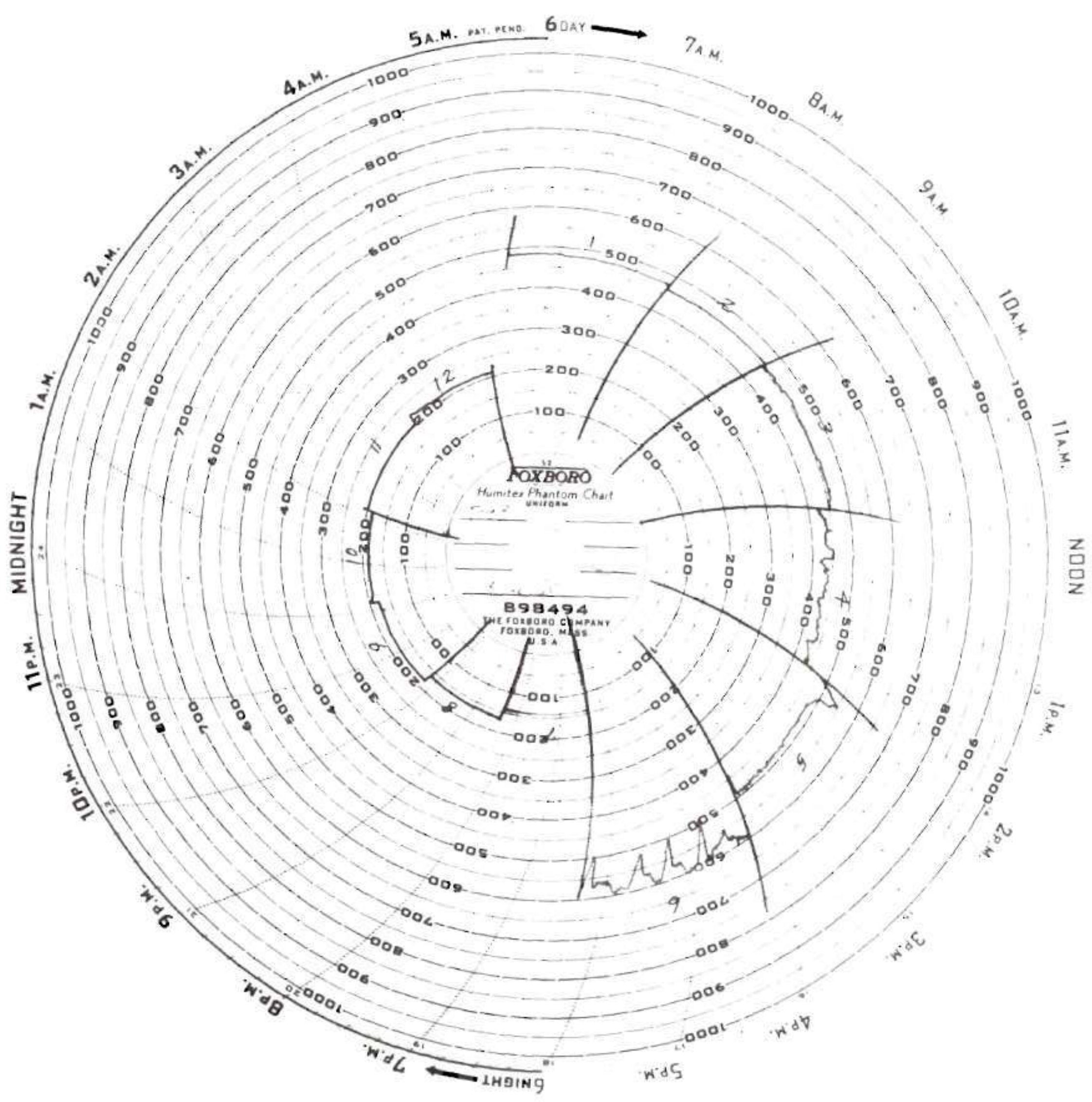


Fig. 15. Vibration Study, Variable Inductance Pickup Method, Strain-time Recording Chart

CHAPTER IV

CONCLUSIONS

These conslusions, arrived at as the result of this research, have to do with the problem of lubrication of the spindle:

1. The thread load seems to affect the lubrication of the spindle only as it affects the Sommerfeld number.

2. Oil whip in general increases the friction over what would be expected with the same oil at a given Sommerfeld number. It also causes the friction produced to be unsteady.

3. Other vibration effects encountered during the research seem to have little effect on the friction.

The following can be concluded about the action of the spindle:

1. There is a belt affected--if not caused--elliptic motion of the end of the spindle which is directed with the major axis at right angles to the belt. This motion is the largest component of the motion of the end of the shaft. Its major frequency is at the running speed of the spindle, but it has higher frequency components.

2. Oil-film whirl does occur at times.

3. Oil whip occurs at speeds well above twice the natural frequency of the spindle. The lowest speed at which it occurs and if it occurs at all is affected by the viscosity of the oil and the load on the bearing. Raising the viscosity lowers the

lowest speed at which it occurs. A higher load increases the chance that it will occur.

4. The natural frequency of the spindle assembly varies from approximately 4800 to 3000 cycles per minute, depending upon the speed of rotation of the spindle and the oil used. The variation in the natural frequency of the spindle assembly is probably due to the change in "flexibility" of the oil film with the change in viscosity and speed. Thus the method used to determine the natural frequency of the shaft seems to be an accurate one.

CHAPTER V

RECOMMENDATIONS

So that it will be known what loads the spindle and bobbin should be designed for, it is recommended that the thread loads encountered in spinning be investigated.

An important simplification made in this research is the elimination of the bobbin and package. Before the results of this research can be applied directly to the actual system, it will be necessary to know: 1. what the limits of the eccentricity of the bobbin with various package loads are and what the effects of this eccentric load in the lubrication and the action of the spindle are, 2. how the bobbin and package load affect the natural frequency of the spindle system, and 3. what the effect of the fact that the thread load is constantly changing direction is (particularly when the rate of speed of this change approaches the natural frequency of the spindle system or a submultiple of the running speed). It is recommended that the above factors be investigated in order to obtain a complete picture of the spindle vibration.

APPENDIX

Technical Data

Spindle

The spindle is a Saco-Lowell McMullan Spindle equipped with either a plain cast-iron bolster with cast iron steps and guides or a plain steel tubing bolster with porous-metal inserts for steps and guides. The plain cast-iron bolster was used.

Spindle Oils

See Table 3 and Fig. 16.

Strain Gages

SR-4 (AB-7) gages.

Gage factor: 2.00 1 per cent

Resistance: 120 0.5 ohms

Lot number: 236

Instrumentation

Strain-time recorder: Baldwin Southwark SR-4 Strain

Recorder

Temperature indicator: Foxboro, Portable Indicator, Model 8106.

"Strobotac:" General Radio Co., Type 631-B.

Cathode-ray oscillograph: Dumont, Model 304 H.

Audio amplifier: Hewlett Packard, Model 200 AB.

Brinell Microscope: Bausch and Lomb, Model UL 1274.

Table 3. Spindle Oils

Identifying Letter	Viscosity in Centistokes at 100°F	130°F	210°F	Company Designation
A	46.38	22.67	6.27	TL-1514
B	39.54	19.70	5.70	TL-1517
C	21.28	11.63	3.87	TL-1513
D	17.13	9.76	3.49	TL-1515
E	13.32	8.40	2.96	TL-1512
F	66.87	31.44	8.04	TL-2301
G	90.20	40.75	9.67	TL-2517
H	150.36	63.40	13.12	TL-2518

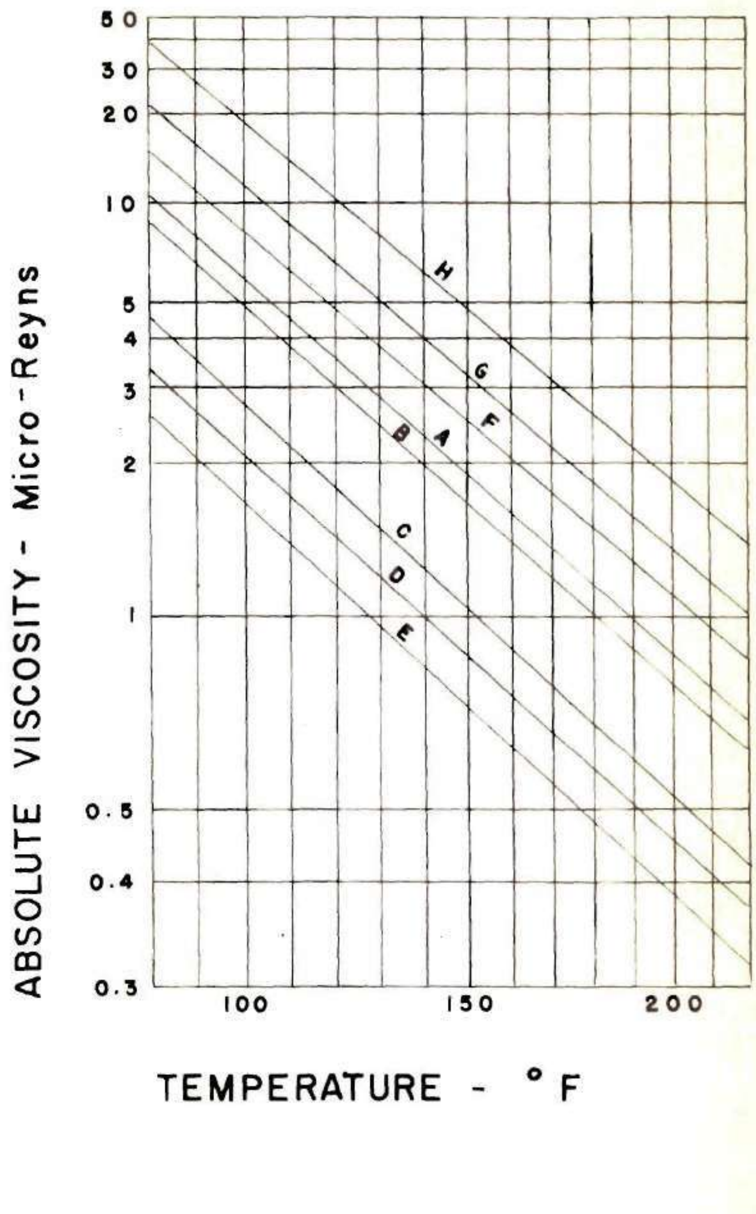


FIG. 16. SPINDLE OILS, VISCOSITY-TEMPERATURE CURVE

Table 4. Calibration of Friction Measuring Apparatus, Tabulated Data.

Calibrating Weights Number	Calibrating Weights Weight gms.	Cumulative Weight gms.	Chart Reading S. L. 2 Run 1	Run 2	Run 3	Run 4	Run 5	Run 6	S. L. 5
1	20.6938	20.6938	150	142	187	181	165	182	71
2	6.1046	26.7984	185	182	238	221	232	228	95
3	6.0544	32.8528	298	308	274	278	278	272	119
4	6.7495	39.6023	358	382	422	418	408	368	172
5	6.6932	46.2955	400	448	482	488	485	485	198
6	6.1916	52.4871	492	512	522	557	528	548	218
7	6.6523	59.1394	530	582	621	618	579	611	239
8	6.1720	65.3114	622	622	650	680	725	687	277
9	6.5937	71.9051	658	690	741	755	728	748	282
10	6.6498	78.5549	732	740	838	830	732	828	286
11	6.5024	85.0573	793	808	870	895	861	882	358
12	6.6810	91.7383	862	860	932	962	939	958	369
	6.8410	98.5793	908	922	-	-	-	-	411
		161.10	-	-					671
		180.00							736
		199.53							828

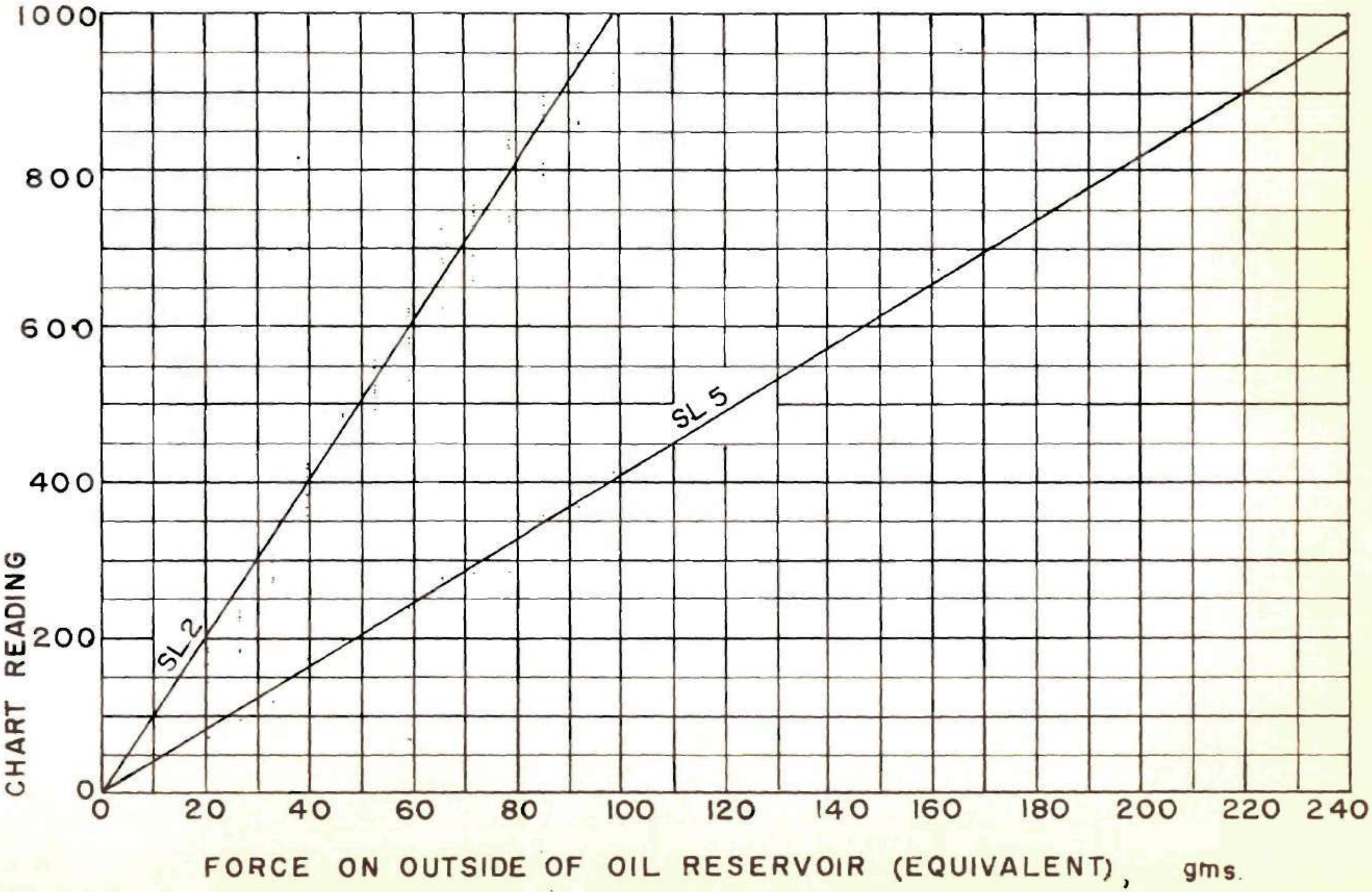


FIG. 17. CALIBRATION OF FRICTION MEASURING APPARATUS, CONVERSION CURVE

Derivations

Load on Belt, Journal Bearing, and Pivot Bearing

Due to cord load.--See Fig. 18 for free bodies.

Summation of moments around 1:

$$\Sigma M_1 = 0 = C\, b - B\, (a + b)$$

or:

$$B = C \frac{b}{(a + b)} \tag{1}$$

Summation of moments around 3:

$$\Sigma M_3 = 0 = L\, e - B\, (e + d)$$

or:

$$L = B \frac{e + d}{e}$$

substituting from (1):

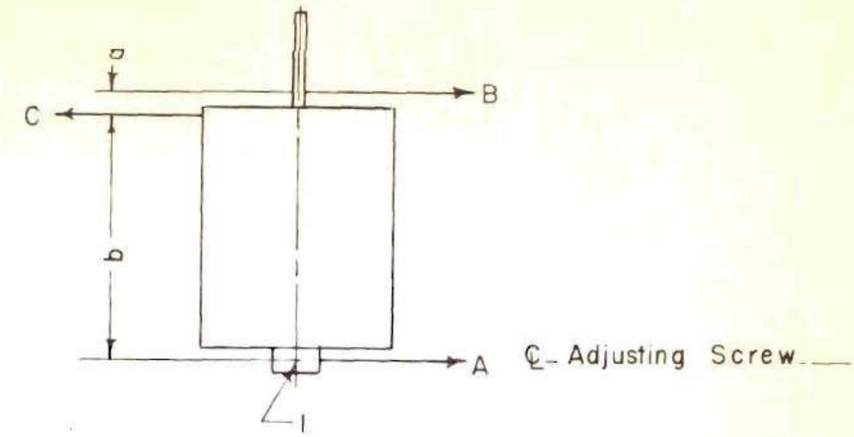
$$L = C \frac{b\, (e + d)}{e\, (a + b)} \tag{2}$$

Summation of moments around 2:

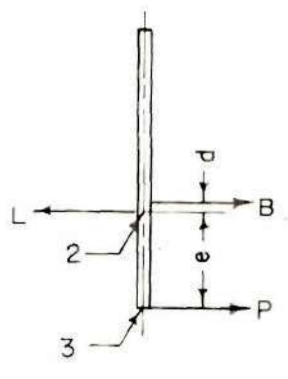
$$\Sigma M_2 = 0 = P\, e - B\, d$$

or:

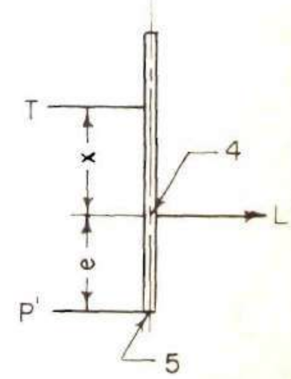
$$P = B \frac{d}{e}$$



(a) FREE BODY OF MOUNTING BASE, OIL RESERVOIR, BOLSTER, AND SPINDLE ASSEMBLY UNDER CORD LOAD



(b) FREE BODY OF SPINDLE UNDER CORD LOAD



(c) FREE BODY OF SPINDLE UNDER THREAD LOAD

FIG. 18. FREE BODIES FOR DETERMINING LOAD ON BEARINGS

substituting from (1):

$$P = C \frac{d\ b}{e\ (a + b)} \tag{3}$$

By field measurement:

$a = 0.75$ in.

$b = 13.65$ in.

$d = 0.20$ in.

$e = 5.13$ in.

Substituting values in (1):

$$B = C \frac{13.65}{(0.75 + 13.65)}$$

$$B = 0.947\ C$$

Substituting values in (2):

$$L = C \frac{13.65\ (5.13 + 0.20)}{5.13\ (0.75 + 13.65)}$$

$$L = 0.985\ C$$

Substituting values in (3):

$$P = C \frac{(0.20)(13.65)}{5.13\ (0.75 + 13.65)}$$

$$P = 0.0370\ C$$

Due to thread load.--

Summation of moments around 5:

$$\Sigma M_5 = T\ (x + e\ ) - L'\ e = 0$$

or:

$$L' = T\ \frac{x + e}{e} \qquad (4)$$

Summation of moments around 4:

$$\Sigma M_4 = 0 = T\ x - P'\ e$$

or:

$$P' = T\ \frac{x}{e} \qquad (5)$$

For values of x see Fig. 8. Substituting these values into (4) and (5):

for position A: $L' = 1.671\ T$; $P' = 0.674\ T$

for position B: $L' = 2.135\ T$; $P' = 1.131\ T$

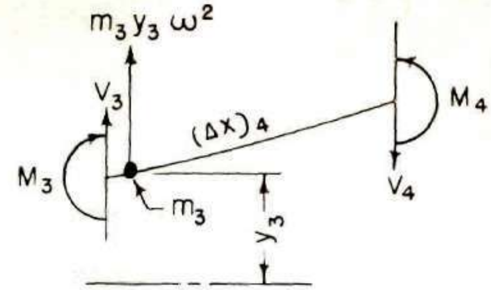
for position C: $L' = 2.645\ T$; $P' = 1.649\ T$

for position D: $L' = 3.122\ T$; $P' = 2.222\ T$

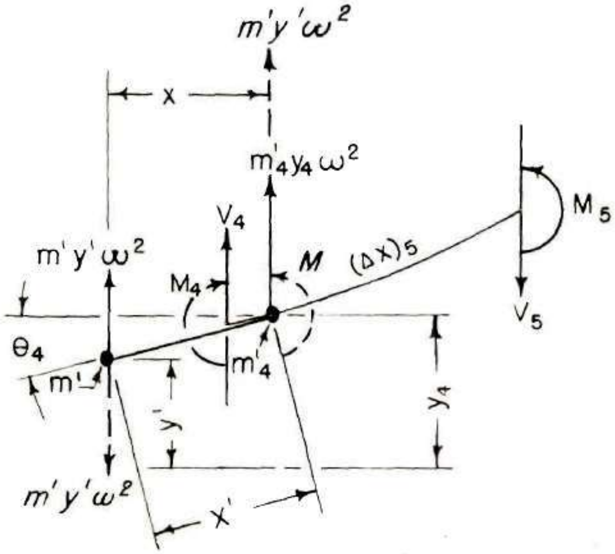
Effect of the Cantilevered Mass in the Equivalent System

Fig. 19 (a) shows the beam section immediately before the cantilevered load. From the free body the following relationships are easily obtained:

$$V_4 = V_3 + m_3\ y_3 \omega^2$$



(a) TYPICAL BEAM SECTION



(b) BEAM SECTION WITH CANTILEVERED WEIGHT

FIG. 19. EFFECT OF CANTILEVERED MASS
FREE BODIES OF BEAM SECTIONS

$$M_4 = M_3 + V_4 (\Delta x)_4$$

Fig. 19 (b) shows the beam section where the cantilevered load is assumed to be point applied. From this free body:

Summation of forces vertically:

$$V_5 = V_4 + m'_4 \, y_4 \omega^2 + m' \, y' \omega^2 \tag{6}$$

The moment just to the right of 4:

$$M_{4R} = M_4 - M \tag{7}$$

Summation of moments around the right end of the section:

$$M_5 = M_{4R} + (\Delta x)_5 (m'_4 \, y_4 \omega^2 + m' \, y' \omega^2) \tag{8}$$

From geometry:

$$y' = y_4 - X' \sin \theta_4 \tag{9}$$

$$x = X' \cos \theta_4 \tag{10}$$

$$M = m' \, y' \omega^2 \, x \tag{11}$$

Substituting (9) into (6) and regrouping:

$$V_5 = V_4 + \omega^2 \, [m'_4 \, y_4 + m' \, (y_4 - X' \sin \theta_4)] \tag{12}$$

Substituting (9) and (11) into (7):

$$M_{4R} = M_4 - m' \omega^2 \, (y_4 - X' \sin \theta_4)(X' \cos \theta_4) \tag{13}$$

Substituting (9) into (8):

$$M_5 = M_{4R} + (\Delta x)_5 [m'_4 \; y_4 \omega^2 + m' \omega^2 \; (y_4 - X' \sin \theta_4)] \tag{14}$$

Since the slope of beams is small (θ_4 is small):

$$\sin \theta_4 \doteq \theta_4 \tag{15}$$

$$\cos \theta_4 \doteq 1 \tag{16}$$

Substituting (15) and (16) into (12), (13), and (14):

$$V_5 = V_4 + \omega^2 \; [m'_4 \; y_4 + m' \; (y_4 - X' \; \theta_4)] \tag{17}$$

$$M_{4R} = M_4 - m' \omega^2 \; X' (y_4 - X' \; \theta_4) \tag{18}$$

$$M_5 = M_{4R} + (\Delta x)_5 \omega^2 \; [m'_4 \; y_4 + m' \; (y_4 - X' \; \theta_4)] \tag{19}$$

It is seen from equation (17) that the effect of the cantilevered moss on the shear is:

$$\omega^2 \; m' \; (y_4 - X' \; \theta_4) \tag{20}$$

It is seen from equation (18) that the moment introduced by the cantilevered mass is:

$$- m' \omega^2 \; X' \; (y_4 - X' \; \theta_4) \tag{21}$$

The extension to four masses m', m'', m''', and m^{iv} is obvious.

Sample Calculations

Determination of Sommerfeld Number and Coefficient of Friction Variable

<u>Sommerfeld</u> <u>number</u>.--

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P}$$

$r = 0.1794$ in.

$c = 0.00315$ in.

N, from experimental data.

μ, from Fig. 16, knowing the temperature from experimental data.

$$P = \frac{L' - L + P' - P}{A} = \frac{W}{A} \quad \text{(see Derivations)}$$

$$A = d\,L$$

$$= (0.365 \text{ in.})(2.14 \text{ in.})$$

$$A = 0.781 \text{ in.}^2$$

Combining known values:

$$S = \left(\frac{0.1794 \text{ in.}}{0.00315 \text{ in.}}\right)^2 \frac{\mu \quad N}{\frac{W}{0.781 \text{ in.}^2}}$$

$$S = (2538 \text{ in}^2) \frac{\mu N}{W}$$

For a sample calculation, the first example with thread load in Table 5 is used (data sheet number T-4,1).

$N = 3990$ rpm, from data.

$t = 90^{o}F$, from data.

From Fig. 16 for oil E at $90^{o}F$

$$\mu = 2.05 \times 10^{-6} \text{ reyns}$$

$$L = 0.985\ C$$

$$P = 0.0370\ C$$

$$L' = 2.135\ T \quad \text{(at position B)}$$

$$P' = 1.131\ T \quad \text{(at Position B)}$$

$$C \text{ (cord loan)} = 0.532 \text{ lbs.}$$

$$T \text{ (thread load)} = 0.396 \text{ lbs.} \quad \text{(at postion B)}$$

Therefore:

$$W = [2.135\ (0.396 \text{ lbs.}) - 0.985\ (0.532 \text{ lbs.})]$$
$$+ [1.131\ (0.396 \text{ lbs.}) - 0.0370\ (0.532 \text{ lbs.})]$$

$$W = 0.746 \text{ lbs.}$$

Therefore:

$$S = (2538 \text{ in}^2)\frac{(2.05 \times 10 \quad \text{reyn})(3990\ 1/\text{min.})}{0.746 \text{ lb.}}$$

$$S = 27.8 \text{ sec/min}$$

<u>Coefficient</u> <u>of</u> <u>Friction</u> <u>Variable</u>.--$\frac{r}{c}$ f

where:

$$f = \text{coefficient of friction} = \frac{F}{W}$$

where:

W is computed as above.

F is obtained by converting the chart reading to grams force of the outside of the oil reservoir (F') by means of Fig. 17 and then converting the force from the outside of the oil reservoir to the inside of the journal by use of the ratio of diameters

$$\frac{0.865\ \text{in.}}{0.365\ \text{in.}} = 2.367$$

Combining known values:

$$\frac{r}{c}\, f = \frac{0.1794\ \text{in.}}{0.00315\ \text{in.}}\ \frac{2.367\ F'}{W}\ \frac{\text{lb}}{453.6\ \text{gm.}}$$

$$\frac{r}{c}\, f = 0.2982\ \frac{\text{lb.}}{\text{gm.}}\ \frac{F'}{W}$$

Using the same example as above:

W = 0.746 lb.   (see above)

F' from Fig. 17 with a chart reading of 288 and a scale length (SL) of 2 is 28.4.

$$\frac{r}{c}\, f = 0.2982\ \frac{\text{lb.}}{\text{gm.}}\ \frac{28.4\ \text{gm.}}{0.746\ \text{lb.}}$$

$$\frac{r}{c}\, f = 11.4$$

Trial in Determination of Natural Frequency

The illustrative trial will be for an assumed speed of 5000 rpm ($\omega = 524$ rad/sec). The method is that described in the original article[13] with the addition of the effect of the cantilevered mass (underscored and overscored on the tabular form) as derived above. The tabular form is shown in Fig. 20.

The boundary conditions at the left end are $y_o = 0$ and $M_o = 0$. Since the values for V_o and θ_o are not known values will be assumed. In part I, V_o is assumed to be one and θ_o, zero; in part II, V_o, zero and θ_o, one. After completion of the two parts it is seen that

$$\theta_3 = 0.000818\ V_o + 1.0162\ \theta_o \tag{22}$$

$$y_3 = 0.001797\ V_o + 5.146\ \theta_o \tag{23}$$

$$M_3 = 5.179\ V_o + 311\ \theta_o \tag{24}$$

From the boundary conditions at m_3 (which over the support)

$$y_3 = 0$$

therefore from (23):

$$V_o = -\ 2863\ \theta_o \tag{25}$$

and substituting (25) into (22) and (24):

$$\theta_3 = -\ 1.327\ \theta_o$$

$$M_3 = -\ 14{,}510\ \theta_o$$

Therefore for the second section of the beam (the cantilevered part) the boundary conditions are (from above) or are assumed to be for part III

	n	ΔV	V	Δx	ΔM	M	$\frac{1}{3}$ M	$\frac{1}{6}$ M	M' M''	βx 10^5	βM' βM''	$\frac{\Delta y'}{\Delta x}$	Δx	Δy'	Δy''	y	mω^2	θ
Part I			1.00								x 10^5	x 10^5		x 10^5		x 10^5		x 10^5
	0	0				0	0	0								0	11.30	0
			1.00	2.03	2.03				0.338 0.667	25.95	8.76 17.58	8.76	2.03	17.79	0			
	1	0.00511				2.030	0.677	0.338								17.79	28.92	
			1.0051	2.03	2.040				1.355 1.695	11.62	15.78 19.70	42.12	2.03	85.5	0			
	2	0.0310				4.070	1.357	0.678								103.3	30.00	
			1.0361	1.07	1.109				2.220 2.404	4.32	9.59 10.39	71.41	1.07	76.4	0			
	3					5.179	1.726	0.863								179.7		81.80
Part II			0								x 10^5	x 10^5		x 10^5		x 1		x 1
	0	0				0	0	0								0	11.30	1.00
			0	2.03	0				0 0	25.95	0 0	0	2.03	0	2.03			
	1	58.6				0	0	0								2.03	28.92	
			58.6	2.03	119.0				19.7 39.7	11.62	229 466	229	2.03	465	2.03			
	2	121.0				119.0	39.7	19.7								4.065	30.00	
			179.6	1.07	192.0				91.5 123.0	4.32	396 531	1091	1.07	1170	1.07			
	3					311.0	103.3	51.8								5.146		1.0162
Part III			1.00								x 10^5	x 10^5		x 10^5		x 10^5		x 10^5
	3	0				0	0	0								0	27.58	0
			1.00	1.63	1.630				0.272 0.543	6.59	1.792 3.579	1.792	1.63	2.92	0			
	4	0.0023				1.630	0.543	0.272								2.92	80.3	5.371
					0.008													
		-0.0048				1.638	0.546	0.273										
			0.9975	1.45	1.446				1.060 1.301	0.586	0.621 0.764	5.992	1.45	8.69	0			
	5	0.0079				3.084	1.028	0.514								11.61	68.3	
			1.0054	2.64	2.655				1.984 2.427	13.40	26.60 32.54	33.36	2.64	88.0	0			
	6	0.0426				5.739	1.913	0.956								99.6	42.7	
			1.0480	2.64	2.761				3.330 3.789	22.12	73.6 83.4	139.5	2.64	368	0			
	7	0.1530				8.500	2.833	1.417								467	32.77	
			1.201	2.64	3.18				4.78 5.31	39.18	187.1 208.0	410.0	2.64	1082	0			
	8	0.210				11.68	3.89	1.95								1549	13.58	
			1.411															
	(9)																	
Part IV			0								x 1	x 1		x 1		x 1		x 1
	3	0				-14,510	-4840	-2420								0	27.58	-1.327
			0	1.63	0				-7260 -7260	6.59	-0.478 -0.478	-0.478	1.63	-0.780	-2.16			
	4	-236.0				-14,510	-4840	-2420								-2.940	80.3	-2.283
					-30													
		-59.5				-14,540	-4850	-2420										
			-296	1.45	-429				-7350 -7410	0.586	-0.043 -0.043	-0.999	1.45	-1.149	-1.92			
	5	-431				-14,970	-4990	-2500								-6.31	68.3	
			-727	2.64	-1920				-7810 -8130	13.40	-1.048 -1.089	-2.090	2.64	-5.51	-3.50			
	6	-655				-16,890	-5630	-2820								-15.32	42.7	
			-1382	2.64	-3650				-9050 -9670	22.12	-2.015 -2.14	-5.19	2.64	-13.70	-3.50			
	7	-1062				-20,540	-6850	-3420								-32.5	32.77	
			-2444	2.64	-6450				-11370 -12450	39.18	-4.45 -4.87	-11.77	2.64	-31.0	-3.50			
	8	-909				-27,090	-9030	-4520								-67.0	13.58	
			-3353															
	(9)																	

Fig. 20. Trial in Determination of the Natural Frequency of the Spindle, Tabular Form

$$V_3 = 1.00$$
$$M_3 = y_3 = \theta_3 = 0$$

and for part IV

$$V_3 = y_3 = 0$$
$$M_3 = -14,510 \quad \text{(dropping the } \theta_o \text{ in the tabular form)}$$
$$\theta_3 = -1.327$$

For the effect of the cantilevered mass, substitute the values in the equivalent system (Table 6) and the following values from the tabular form into equations (20) and (21):

$$\omega^2 = 2.74 \times 10^5$$

in part III

$$y_4 = 2.92 \times 10^{-5}\, V_3$$
$$\theta_4 = 5.371 \times 10^{-5}\, V_3$$

in part IV

$$y_4 = -2.940\, \theta_3$$
$$\theta_4 = -2.283\, \theta_o$$

The effect of the cantilevered mass in part III

$$\text{effect on } V = -480 \times 10^{-5}$$
$$\text{effect on } M = -764 \times 10^{-5}$$

in part IV

$$\text{effect on } V = -59.5$$

$$\text{effect on } M = 32.5$$

After completion of the last two parts it is seen that

$$V_9 = 1.411\ V_3 - 3353\ \theta_o$$
$$M_8 = 11.68\ V_3 - 27{,}090\ \theta_o$$

Both V_9 and M_8 should be equal to zero. However this condition will be true only at the critical speed. Therefore it is arbitrarily assigned that V_9 is equal to zero. Therefore

$$V_3 = +2375\ \theta_o$$

and

$$M_8 = +560\ \theta_o$$

This value is the excess bending moment. A graph of the excess bending moment (with θ_o arbitrarily assumed equal to one) versus the assumed speed locates the critical speeds-- where the excess bending moment equals zero (see Fig. 10).

Determination of the Frequency of Vibration from the Oscillograph

a is the scale length of one cycle of the running speed vibration as shown on the oscillograph.

f is the frequency of the running speed vibration as read from the audio oscillator.

n is the number of cycles of the natural frequency vibration per second.

L units scale length

Then:

$$F_c = \frac{f}{a} \times \frac{n}{L}$$

Example: A, Table 10

$$a = 18 \text{ units}$$

$$f = 10,880 \text{ cyc/min}$$

$$L = 8.5 \text{ units}$$

$$n = 6 \text{ cycles}$$

Therefore

$$F_c = \frac{10,880}{18} \times \frac{6}{8.5}$$

$$F_c = 4260 \text{ cyc/min}$$

Table 5. Spindle Bearing Friction, Effect of Thread Load, Tabulated Data

Speed	Oil	Journal Temp.	Cord Load	Thread Weight	Load Posi-tion	Total Radial Load	Sommerfeld Number	Chart Reading	Scale Length	Coef. of Friction Variable
rpm		^{o}F	lbs.	lbs.		lbs.	sec/min			
Series I										
4000	E	89	0.532	0	-	0.547	38.8	262	2	14.2
4000	E	90	1.036	0	-	1.062	19.58	292	2	8.15
4000	E	90	1.531	0	-	1.571	13.21	393	2	7.36
4000	E	91	2.036	0	-	2.082	9.74	398	2	5.62
4010	E	91	2.531	0	-	2.598	7.84	402	2	4.58
4010	E	92	3.035	0	-	3.194	6.21	482	2	4.46
4040	E	95	3.528	0	-	3.618	5.16	543	2	4.42
3990	E	90	0.532	0.396	B	0.746	27.8	288	2	11.4
4010	E	92	0.532	0.800	B	2.101	9.42	410	2	5.75
								(305)		(4.27)
4010	E	93	0.532	1.197	B	3.354	5.78	535	2	4.70
4010	E	96	0.532	1.601	B	4.682	3.78	778	2	4.82
4010	E	90	1.036	0.396	B	0.583	35.8	314	2	15.9
4010	E	90	1.036	0.800	B	1.236	16.88	318	2	7.60
4010	E	92	1.036	1.197	B	2.839	7.00	514	2	5.34
4010	E	95	1.036	1.601	B	4.167	4.43	770	2	5.44
								(690)		(4.86)
4010	E	90	0.532	0.396	C	1.154	18.08	383	2	9.80
4020	E	90	0.532	0.800	C	2.879	7.26	435	2	4.45
4020	E	95	0.532	1.197	C	4.585	4.05	771	2	4.96
4020	E	90	1.036	0.396	C	0.639	32.6	398	2	17.9
4020	E	90	1.036	0.800	C	2.374	8.81	412	2	5.12
4020	E	94	1.036	1.197	C	4.070	4.66	620	2	4.49

Table 5. (continued) Spindle Bearing Friction, Effect of Thread Load, Tabulated Data

Speed	Oil	Journal Temp.	Cord Load	Thread Weight	Load Position	Total Radial Load	Sommerfeld Number	Chart Reading	Scale Length	Coef. of Friction Variable
rpm		°F	lbs.	lbs.		lbs.	sec/min			
4000	E	90	0.532	0.396	A	0.480	43.3	285	2	17.6
4000	E	90	0.532	0.800	A	1.330	15.61	289	2	6.60
4000	E	90	0.532	1.197	A	2.259	8.04	299	2	3.92
4000	E	90	0.532	1.601	A	3.213	6.46	396	2	3.62
4000	E	90	1.036	0.396	A	0.587	35.4	300	2	15.1
4000	E	90	1.036	0.800	A	0.815	25.5	370	2	13.4
4000	E	92	1.036	1.197	A	1.744	11.31	380	2	6.41
4000	E	92	1.036	1.601	A	2.698	7.34	388	2	4.24
4000	E	90	0.532	0.396	D	1.565	13.29	304	2	5.74
3990	E	94	0.532	0.800	D	3.729	5.04	513	2	4.06
4000	E	90	1.036	0.396	D	1.050	19.80	296	2	8.30
4000	E	94	1.036	0.800	D	3.214	5.86	403	2	3.70
3980	E	94	0.532	1.197	D	5.846	3.21	286	5	3.58
3990	E	95	1.036	1.197	D	5.331	3.46	304	5	4.20
3960	E	103	0.532	1.601	D	8.013	1.93	525	5	4.79
3990	E	95	0.532	1.601	C	6.34	2.90	262	5	3.03
		(97)					(2.79)	(491)		(5.66)
3990	E	100	1.036	1.601	C	5.82	2.86	510	5	6.40
Series II										
6190	A	104	0.532	0	-	0.547	151.9	947	2	50.3
6260	A	104	1.036	0	-	1.062	79.1	803	2	22.2
6280	A	104	1.531	0	-	1.571	53.7	884	2	15.6
6290	A	99	2.036	0	-	2.082	46.3	647	2	9.10
								(712)		(10.1)

Table 5. (continued) Spindle Bearing Friction, Effect of Thread Load, Tabulated Data

Speed rpm	Oil	Journal Temp. °F	Cord Load lbs.	Thread Weight lbs.	Load Position	Total Radial Load lbs.	Sommerfeld Number sec/min	Chart Reading	Scale Length	Coef. of Friction Variable
6300	A	100	2.531	0	-	2.598	35.3	729	2	8.25
6300	A	101	3.035	0	-	3.104	29.5	847	2	8.00
								(860)		(8.10)
								(892)		(8.26)
6300	A	105	3.530	0	-	3.618	22.8	892	2	7.10
6300	A	106	4.033	0	-	4.133	19.32	898	2	6.39
6220	A	104	0.532	0	-	0.547	152.8	901	2	48.5
6330	A	108	4.528	0	-	4.64	16.41	422	5	6.65
								(436)		(6.85)
6330	A	109	5.031	0	-	5.15	14.33	434	5	6.16
6330	A	111	5.526	0	-	5.66	12.47	477	5	6.14
6340	A	115	6.031	0	-	6.18	10.40	496	5	5.96
6340	A	119	6.524	0	-	6.69	8.71	541	5	5.93
		(118)					(8.89)	(524)		(5.74)
6330	A	120	7.029	0	-	7.20	7.91	556	5	5.64
6330	A	126	7.524	0	-	7.71	6.42	643	5	6.11
6260	A	105	0.532	0.396	A	0.380	215	347	5	74.9
6290	A	105	0.532	0.800	A	1.330	61.7	350	5	19.3
6300	A	105	0.532	1.197	A	2.259	36.4	304	5	9.91
6300	A	108	0.532	1.601	A	3.213	23.6	387	5	8.82
6290	A	105	1.036	0.396	A	0.587	140.0	350	5	43.6
6300	A	105	1.036	0.800	A	0.815	101.0	368	5	33.0
6310	A	105	1.036	1.197	A	1.644	50.1	370	5	16.5
6310	A	106	1.036	1.601	A	2.698	29.6	385	5	10.5
6310	A	105	2.036	0.396	A	1.489	55.4	307	5	15.1

Table 5. (continued) Spindle Bearing Friction, Effect of Thread Load, Tabulated Data

Speed rpm	Oil	Journal Temp. °F	Cord Load lbs.	Thread Weight lbs.	Load Position	Total Radial Load lbs.	Sommerfeld Number sec/min	Chart Reading	Scale Length	Coef. of Friction Variable
6310	A	105	2.036	0.800	A	1.123	73.4	304	5	19.9
6320	A	105	2.036	1.197	A	0.728	113.2	325	5	32.8
6320	A	105	2.036	1.601	A	1.678	49.5	359	5	14.8
6310	A	99	3.035	0.396	A	2.493	38.8	304	5	8.99
6310	A	103	3.035	0.800	A	2.087	41.8	347	5	12.2
6310	A	104	3.035	1.197	A	1.692	50.0	361	5	15.7
6310	A	105	3.035	1.601	A	1.286	64.1	377	5	21.4
6310	A	101	4.033	0.396	A	3.427	26.8	342	5	7.33
6310	A	100	4.033	0.800	A	3.018	31.4	331	5	8.05
6310	A	104	4.033	1.197	A	2.623	32.3	384	5	10.7
6300	A	105	4.033	1.601	A	2.223	37.0	396	5	13.1
6310	A	103	5.031	0.396	A	4.79	18.22	348	5	5.29
6310	A	99	5.031	0.800	A	3.95	24.5	326	5	6.04
6310	A	104	5.031	1.197	A	3.56	23.8	383	5	7.92
6310	A	105	5.031	1.601	A	3.15	26.2	398	5	9.31
6310	A	105	5.526	0.396	A	4.83	17.08	399	5	6.08
6310	A	106	5.526	0.800	A	4.42	18.11	410	5	6.81
6300	A	103	5.526	1.197	A	4.03	21.6	394	5	7.20
6300	A	105	5.526	1.601	A	3.62	22.7	399	5	8.11
6260	A	105	0.532	0.396	B	0.746	109.8	387	5	38.2
6290	A	105	0.532	0.800	B	2.066	39.6	384	5	13.7
6300	A	107	0.532	1.197	B	3.354	23.4	390	5	8.59
6300	A	109	0.532	1.601	B	3.672	20.0	398	5	7.99
6290	A	106	1.036	0.396	B	0.583	136.8	362	5	41.9
6300	A	105	1.036	0.800	B	1.551	53.0	378	5	17.8
6310	A	106	1.036	1.197	B	2.839	28.2	385	5	10.0

Table 5. (continued) Spindle Bearing Friction, Effect of Thread Load, Tabulated Data

Speed rpm	Oil	Journal Temp. °F	Cord Load lbs.	Thread Weight lbs.	Load Position	Total Radial Load lbs.	Sommerfeld Number sec/min	Chart Reading	Scale Length	Coef. of Friction Variable
6310	A	108	1.036	1.601	B	4.168	18.21	397	5	7.00
6310	A	107	2.036	0.396	B	1.525	51.4	377	5	18.1
6310	A	107	2.036	0.800	B	1.119	70.1	383	5	25.2
6310	A	108	2.036	1.197	B	1.819	66.2	389	5	15.8
6310	A	108	2.036	1.601	B	3.147	24.2	392	5	9.17
6310	A	106	3.035	0.396	B	2.489	31.1	398	5	11.8
6310	A	105	3.035	0.800	B	2.083	39.5	398	5	14.1
6310	A	106	3.035	1.197	B	1.687	47.4	401	5	17.5
6310	A	108	3.035	1.601	B	2.125	35.7	407	5	14.1
6310	A	106	4.033	0.396	B	3.420	23.4	400	5	8.66
6310	A	106	4.033	0.800	B	3.014	26.6	400	5	9.79
6310	A	106	4.033	1.197	B	2.618	30.6	399	5	11.2
6310	A	108	4.033	1.601	B	2.210	34.4	408	5	13.6
6290	A	104	0.532	0.396	C	1.155	73.1	398	5	25.4
6300	A	106	0.532	0.800	C	2.889	27.4	405	5	10.3
6310	A	108	0.532	1.197	C	4.585	16.60	411	5	6.59
6310	A	110	0.532	1.601	C	5.989	12.02	443	5	5.43
6300	A	105	1.036	0.396	C	0.639	128.9	385	5	44.4
6310	A	105	1.036	0.800	C	2.374	34.7	390	5	12.1
6320	A	108	1.036	1.197	C	4.070	18.70	409	5	7.38
6320	A	110	1.036	1.601	C	5.82	12.40	416	5	5.22
6310	A	104	2.036	0.396	C	1.525	55.6	380	5	18.3
6300	A	108	2.036	0.800	C	1.354	56.0	392	5	21.3
6300	A	107	2.036	1.197	C	3.050	25.6	399	5	9.63
6310	A	110	2.036	1.601	C	4.80	15.00	410	5	6.27
6300	A	107	3.035	0.396	C	2.489	32.4	385	5	11.4

Table 5. (continued) Spindle Bearing Friction, Effect of Thread Load, Tabulated Data

Speed rpm	Oil	Journal Temp. °F	Cord Load lbs.	Thread Weight lbs.	Load Position	Total Radial Load lbs.	Sommerfeld Number sec/min	Chart Reading	Scale Length	Coef. of Friction Variable
6300	A	107	3.035	0.800	C	2.086	37.5	390	5	13.8
6300	A	107	3.035	1.197	C	2.028	38.6	399	5	14.5
6300	A	110	3.035	1.601	C	3.77	19.08	402	5	7.85
6300	A	108	3.530	0.396	C	2.945	25.7	391	5	9.78
6300	A	108	3.530	0.800	C	2.542	29.8	392	5	11.4
6300	A	108	3.530	1.197	C	2.156	35.2	402	5	13.8
6300	A	110	3.530	1.601	C	3.26	22.0	402	5	9.09
6250	A	105	0.532	0.396	D	1.665	49.0	391	5	17.3
6260	A	105	0.532	0.800	D	3.729	21.9	411	5	8.10
6270	A	109	0.532	1.197	D	5.848	12.52	405	5	5.10
6280	A	110	0.532	1.601	D	8.01	8.94	431	5	3.94
6260	A	106	1.036	0.396	D	1.050	75.5	380	5	26.6
6270	A	107	1.036	0.800	D	3.214	24.2	394	5	9.04
6270	A	108	1.036	1.197	D	5.331	14.18	409	5	5.99
6270	A	110	1.036	1.601	D	7.50	9.54	409	5	4.26
6270	A	106	1.531	0.396	D	1.099	72.4	379	5	25.2
6270	A	105	1.531	0.800	D	2.705	30.2	399	5	11.8
6270	A	107	1.531	1.197	D	4.822	16.18	411	5	6.25
6270	A	110	1.531	1.601	D	6.99	10.24	428	5	4.48
6270	A	106	2.036	0.396	D	1.568	50.8	381	5	17.9
6270	A	107	2.036	0.800	D	2.194	35.5	406	5	13.6
6280	A	107	2.036	1.197	D	4.111	18.98	416	5	7.37
6280	A	107	2.036	1.601	D	6.48	12.03	428	5	4.82
6280	A	105	2.531	0.396	D	2.048	40.0	387	5	13.9
6280	A	105	2.531	0.800	D	1.682	48.7	408	5	17.8
6280	A	106	2.531	1.197	D	3.795	21.0	417	5	8.00

Table 5. (continued) Spindle Bearing Friction, Effect of Thread Load, Tabulated Data

Speed rpm	Oil	Journal Temp. °F	Cord Load lbs.	Thread Weight lbs.	Load Position	Total Radial Load lbs.	Sommerfeld Number sec/min	Chart Reading	Scale Length	Coef. of Friction Variable
6280	A	109	2.531	1.601	D	5.96	12.27	401	5	4.95
6250	A	104	0.532	0	-	0.547	153.2	316	5	42.2
6290	A	104	1.036	0	-	1.062	79.4	332	5	23.0
6310	A	99	2.036	0	-	2.082	46.4	281	5	9.89
6310	A	100	3.035	0	-	3.104	30.4	322	5	7.61
6310	A	105	4.033	0	-	4.133	19.93	385	5	6.88
6310	A	110	5.031	0	-	5.15	13.99	456	5	6.49
6310	A	115	6.031	0	-	6.18	11.37	524	5	6.20
6310	A	116	6.526	0	-	6.69	9.33	538	5	5.88
6310	A	118	7.029	0	-	7.20	8.22	567	5	5.75
6310	A	120	7.524	0	-	7.71	7.36	578	5	5.46
6320	A	125	8.021	0	-	8.21	6.15	649	5	5.77
		(129)					(5.56)	(671)		(5.98)
6330	A	108	5.031	0.396	C	4.36	17.48	394	5	6.65
6340	A	108	5.031	0.800	C	3.95	19.30	408	5	7.60
6340	A	107	5.031	1.197	C	3.56	22.1	408	5	8.44
6330	A	108	5.031	1.601	C	3.15	24.2	421	5	9.79
6330	A	104	7.029	0.396	C	6.25	13.21	358	5	4.19
6330	A	105	7.029	0.800	C	5.84	14.12	383	5	4.84
6330	A	108	7.029	1.197	C	5.45	13.98	402	5	5.44
6340	A	108	7.029	1.601	C	4.04	18.90	412	5	7.46
Series III										
4000	A	101	0.532	0.396	C	1.154	57.5	311	5	19.79
4000	A	99	0.532	0.800	C	2.889	21.2	324	5	8.24

Table 5. (continued) Spindle Bearing Friction, Effect of Thread Load, Tabulated Data

Speed rpm	Oil	Journal Temp. °F	Cord Load lbs.	Thread Weight lbs.	Load Position	Total Radial Load lbs.	Sommerfeld Number sec/min	Chart Reading	Scale Length	Coef. of Friction Variable
4000	A	100	0.532	1.197	C	4.585	13.08	340	5	5.45
4010	A	100	0.532	1.601	C	6.34	9.45	381	5	4.41
4000	A	98	1.036	0.396	C	0.639	98.4	326	5	37.4
4010	A	97	1.036	0.800	C	3.374	19.26	332	5	7.22
4010	A	97	1.036	1.197	C	4.070	16.38	339	5	6.09
4010	A	99	1.036	1.601	C	5.82	10.57	383	5	4.85
4010	A	99	2.036	0.396	C	1.525	40.3	324	5	15.1
4010	A	99	2.036	0.800	C	1.354	45.5	333	5	18.1
4010	A	99	2.036	1.197	C	3.050	20.2	333	5	8.04
4010	A	100	2.036	1.601	C	4.80	12.69	362	5	5.54
4010	A	99	4.033	0.396	C	3.420	18.00	327	5	7.01
4010	A	99	4.033	0.800	C	3.017	20.4	328	5	7.95
4010	A	99	4.033	1.197	C	2.625	23.4	358	5	9.96
4010	A	100	4.033	1.601	C	2.74	21.9	390	5	10.4
4020	A	100	7.029	0.396	C	6.25	9.61	410	5	4.82
								(330)		(3.97)
4020	A	97	7.029	0.800	C	5.48	11.18	330	5	4.15
4020	A	97	7.029	1.197	C	5.45	11.97	362	5	4.88
		(100)					(11.01)	(890)		(12.0)
Series IV										
4790	E	95	0.532	0.	-	0.547	40.4	158	5	21.2
4790	E	93	0.532	0.396	C	1.154	19.98	155	5	9.86
4790	E	95	0.532	0.800	C	2.889	7.64	182	5	4.64
4790	E	97	0.532	1.197	C	4.585	4.53	235	5	3.77
4790	E	102	0.532	1.601	C	6.34	3.02	343	5	3.97

Table 6. Values for Equivalent System

Masses		Shaft Constants		Distances	
Mass	Value lb sec^2/in x 10^5	Length	Value 1/lb in x 10^5	Length	Value in
m_0	4.12	β_1	25.95	Δx_1	2.03
m_1	10.56	β_2	11.62	Δx_2	2.03
m_2	10.95	β_3	4.32	Δx_3	1.07
m_3	10.07	β_4	6.59	Δx_4	1.63
m'_4	29.29	β_5	0.586	Δx_5	1.45
m_5	24.92	β_6	13.40	Δx_6	2.64
m_6	15.60	β_7	22.12	Δx_7	2.64
m_7	11.95	β_8	39.18	Δx_8	2.64
m_8	4.95			$\Delta x'$	0.40
m'	14.00			$\Delta x''$	0.80
m''	10.52			$\Delta x'''$	1.38
m'''	21.12			Δx^{iv}	1.90
m^{iv}	9.94				

Table 7. Determination of the Natural Frequency of the Spindle, Values of Excess Moment

Assumed Speed rpm	Excess Moment lb.-in.
1000	- 13,710
3000	- 7,550
5000	+ 560
7000	+ 7,770
9000	+ 13,580
13000	+ 21,000

Table 8. Vibration Study, Visual Method

Spindle Bearing Friction, Tabulated Data

Speed rpm	Oil	Temp. °F	Cord Load lbs.	Sommerfeld Number sec/min	Chart Reading	Scale Length	Coef. of Friction Variable	Visual Observations [a]	Chart Remarks [b]
3910	A	95	0.532	123.1	312	5	41.9	A, I, 1.5	-
4880	A	100	0.532	133.3	342	5	45.9	A, I, 1.5	-
5840	A	103	0.532	147.4	347	5	46.4	B,II, 4	-
6800	A	106	0.532	157.8	342	5	45.9	C,II, 4	sr
7850	A	107	0.532	178.3	330	5	44.2	C, 4	sr
8840	A	108	0.532	194.8	310	5	41.6	C, 3	vsr
9800	A	112	0.532	193.1	325	5	43.6	C, IV, 4.5	r
10640	A	119	0.532	179.2	385	5	51.9	C, III, 3	r 10
12000	A	128	0.532	163.0	425	5	57.0	C, III, 3	r 30, su
3950	A	101	3.035	18.5	280	5	6.61	A, II, 2	-
4920	A	100	3.035	23.7	290	5	6.85	A, II, 1.5	-
5880	A	100	3.035	28.3	300	5	7.10	A, II,3	su
6890	A	105	3.035	28.9	320	5	7.57	C, 3	vsr
7900	A	106	3.035	32.2	325	5	7.68	C, 3	-
8900	A	109	3.035	33.4	330	5	7.79	C	-
9890	A	115	3.035	32.2	380	5	7.96	C, 3	sr, su
10900	A	153	5.031	9.60	590	5	8.39	C, 1.5	r 40, vu
					(700)		(9.95)		
					(750)		(10.64)		
11900	A	140	5.031	13.37	520	5	7.40	C, 1.25	r 20
3950	A	107	5.031	9.53	412	5	6.38	A, I, 1	-
4910	A	106	5.031	12.09	438	5	6.79	A, I, 1	-
5900	A	118	7.029	7.69	558	5	5.66	A, I, 1	sr
6880	A	118	7.029	8.95	489	5	4.95	A, I, 1	vsr

Table 8. (continued) Vibration Study, Visual Method

Spindle Bearing Friction, Tabulated Data

Speed rpm	Oil	Temp. °F	Cord Load lbs.	Sommerfeld Number sec/min	Chart Reading	Scale Length	Coef. of Friction Variable	Visual [a] Observations	Chart [b] Remarks
7890	A	123	7.029	9.12	490	5	4.95	A, I, 1.5	-
8910	A	125	7.029	9.89	485	5	4.93	A, I, 1	u
9900	A	134	7.029	9.04	517	5	5.24	B, 2	r 50
10900	A	145	7.029	7.95	565	5	5.74	B, 1.5	r 40
11910	A	-	7.029	-	610	5	6.20	B, 1.25	r 30, u
					(710)		(7.22)		
3940	H	100	3.035	61.0	488	5	11.48	A, II, 1.5	-
4840	H	105	3.035	63.1	505	5	11.89	A, II, 2	vsu
5760	H	106	3.035	73.7	500	5	11.78	A, II, 2	su
6730	H	110	3.035	76.6	479	5	11.22	A, II, 1.5	vsu
7650	H	116	3.035	72.9	590	5	13.89	A, II, 2	vu
					(490)		(11.50)	A, II, 2	
8590	H	119	3.035	75.5	462	5	10.89	A, II, 2	vsu
9540	H	119	3.035	83.9	390	5	9.25	B, II, 2.5	sr, u
					(420)		(9.89)		
10460	H	130	3.035	67.4	545	5	11.88	C, III,3	r 20
11420	H	154	3.035	46.6	680	5	15.99	C,III, 2.5	r 40
		(145)		(50.4)	(515)		(12.12)	(C, III, 2.5)	(r 15,vu)
					(605)		(14.28)		
3920	E	89	1.531	13.16	156	5	7.25	A, II, 3	-
4800	E	89	1.531	16.00	151	5	7.06	A, II, 1	-
5690	E	90	1.531	18.80	172	5	8.01	A, II, 1.5	-
6590	E	95	1.531	19.65	183	5	8.55	B, II, 3	-
7540	E	95	1.531	22.5	200	5	9.32	C, 3	-
8460	E	97	1.531	24.3	216	5	10.08	C, II, 3	-
9390	E	99	1.531	25.4	219	5	10.21	C, III, 4	sr

Table 8. (continued) Vibration Study, Visual Method

Spindle Bearing Friction, Tabulated Data

Speed rpm	Oil	Temp °F	Cord Load lbs.	Sommerfeld Number sec/min	Chart Reading	Scale Length	Coef. of Friction Variable	Visual [a] Observations	Chart [b] Remarks
10300	E	104	1.531	25.0	248	5	11.54	C,III, 3.5	-
11210	E	10[illegible]	1.531	26.4	251	5	11.92	C, II, 2	sr

Remarks:

[a] Letters designate the number of "dots" that can be seen when the "Strobotac" is flashing with a frequency equal to the speed of the spindle. The number of "dots" tends to indicate the "unsteadiness" of the motion (except for motion type III). The meanings of the symbols are

A approximately 1 to 3 dots
B approximately 4 to 6 dots
C over 6 dots, usually found in a circular cluster

The Roman numerals indicate the type of orbit in which the end of the spindle --the "dots"--tends to travel.

I approximately circular

II approximately elliptic
III  type C (indicates oil whip)
IV circle or ellipse of motion of the end of the spindle filled with "dots"

The Arabic numerals indicate the length of the major axis of the ellipse (or diameter of the circle) in divisions. These numbers are relative values only.

Any of the above marks may be omitted.

[b] The chart remarks are illustrated where possible in Figs. 14 and 15.

Major marks

r indicates a ripple in the chart trace (Fig. 14, run 7).
u indicates an unsteadiness in the chart trace (Fig. 14, run 9; also see below).

Prefix marks

s slight
vs very slight
v very or much

Numbers

Numbers indicate the approximate amplitude of the ripple (when it is large enough to measure) in the chart trace in the value of chart reading (Fig. 14, run 9).

Table 9. Vibration Study, Variable Inductance Pickup Method,

Spindle Bearing Friction Curve, Tabulated Date

Speed	Oil	Temp.	Cord Load	Sommerfeld Number	Chart Reading	Scale Length	Coef. of Friction Variable	Oscillograph Observations Type[a]	Oscillograph Observations Remarks[b]	Chart[c] Remarks
rpm		°F	lbs.	sec/min						
3940	E	90	1.531	13.03	380	2	7.11	II II	vsv vsv	-
4750	E	90	1.531	15.71	391	2	7.35	I I	sv sv	-
5780	E	91	1.531	18.61	427	2	8.00	I I	v v	-
6710	E	94	1.531	20.6	508	2	9.52	I I	- v	-
7690	E	96	1.531	21.6	498	2	9.33	I I	- -	vsr
8650	E	99	1.531	23.4	560	2	10.5	I I	- v	sr
9600	E	102	1.531	24.4	574	2	10.8	I I	- -	sr
10550	E	104	1.531	25.5	664	2	12.4	I I	- vsv	sr
11510	E	108	1.531	26.0	638	2	12.9	I I	- sv	sr
3950	A	100	3.035	19.00	292	5	5.70	I -	v -	-
4860	A	99	3.035	24.0	315	5	7.42	- I	- -	-
5800	A	104	3.035	25.1	353	5	8.35	I I	v v	-

Table 9. (continued) Vibration Study, Variable Inductance Pickup Method,

Spindle Bearing Friction Curve, Tabulated Data

Speed rpm	Oil	Temp. °F	Cord Load lbs.	Sommerfeld Number sec/min	Chart Reading	Scale Length	Coef. of Friction Variable	Oscillograph Observations Type[a]	 Remarks[b]	Chart[c] Remarks
6710	A	104	3.035	29.0	377	5	8.90	V II	v, o v, o	-
7690	A	105	3.035	32.2	385 (370)	5	9.13 (8.72)	II I	v v	sr
8650	A	110	3.035	31.7	382 (368)	5	8.95 (8.65)	II I	v v	sr
9600	A	127	3.035	23.6	465 (510)	5	10.0 (12.0)	III III	v v	r 10
10580	A	132	5.031	14.01	475	5	6.75	III III	v A v	u
11530	A	134	5.031	14.54	510	5	7.24	III VI	- - B	r 10, u
3950	A	105	5.031	10.01	385	5	5.51	I Iv	sv sv	-
4860	A	104	5.031	12.70	395	5	5.65	I IV	v -	-
5790	A	115	7.029	8.15	410	5	4.18	- IV	vv -	-
6730	A	114	7.029	9.60	445	5	4.52	V IV	vv -	sr
7700	A	115	7.029	11.85	455	5	4.62	I IV	vv -	su
8660	A	125	7.029	9.61	516	5	5.24	V I	- -	sr
9630	A	128	7.029	9.89	516	5	5.24	V I	- -	vu

Table 9. (continued) Vibration Study, Variable Inductance Pickup Method,

Spindle Bearing Friction Curve, Tabulated Data

Speed rpm	Oil	Temp. °F	Cord Load lbs.	Sommerfeld Number sec/min	Chart Reading	Scale Length	Coef. of Friction Variable	Oscillograph Observations Type[a]	 Remarks[b]		Chart[c] Remarks
10590	A	141	7.029	8.26	600	5	6.10	IV III	- -	C	r 10, vu
115 0	A	152	7.029	6.46	600	5	6.10	IV III	- -	D	r, vu
3860	A	90	1.036	69.6	228	5	15.7	I I	c sc		-
4860	A	90	1.036	87.6	241	5	16.6	I I	- sv		-
5660	A	94	1.036	94.4	235	5	16.2	I I	- v		sr
6580	A	94	1.036	109.8	248	5	17.0	I I	sv v		-
7540	A	99	1.036	108.8	275	5	19.0	III IV	v vv		r 10
8460	A	101	1.036	116.0	303	5	20.8	IV IV	v vv		r 20
9400	A	103	1.036	122.1	295	5	20.4	III V	- vv	E	sr
10300	A	125	1.036	77.4	445	5	30.6	III VI	- sv	F	r 10
11200	A	141	1.036	59.2	520	5	35.8	IV III	sv sv	G	r 20
3940	H	100	3.035	61.0	430	5	10.1	II II	o -		-
4830	H	103	3.035	66.9	437 (467)	5	11.4 (11.0)	II II	v sv, o		-

Table 9. (continued) Vibration Study, Variable Inductance Pickup Method,

Spindle Bearing Friction Curve, Tabulated Data

Speed	Oil	Temp.	Cord Load	Sommerfeld Number	Chart Reading	Scale Length	Coef. of Friction Variable	Oscillograph Observations Type[a]	Remarks[b]	Chart[c] Remarks
rpm		°F	lbs.	sec/min						
5740	H	105	3.035	74.8	472 (455)	5	11.1 (10.7)	VI IV	v, o sv,o	-
6650	H	109	3.035	77.0	478	5	11.2	IV VI	v, o v	-
7600	H	111	3.035	85.4	462	5	10.9	III III	v, sc, o v, o	-
8560	H	115	3.035	83.7	455	5	10.7	II III	v H -	sr
9500	H	114	3.035	95.3	430	5	10.1	III III	v J sv	u
10450	H	125	3.035	77.5	485	5	11.4	III IV	sv K -	sr
11390	H	140	3.035	56.6	570	5	13.4	III IV	sv L -	vu
3940	E	89	0.532	38.2	141	5	19.0	III III	o o	-
4810	E	89	0.532	46.6	162	5	21.8	III III	o o	-
5700	E	90	0.532	55.2	179	5	23.9	II III	o oo	-
6630	E	93	0.532	58.4	185	5	24.8	III III	o o	sr
7570	E	93	0.532	66.7	200	5	26.8	III III	v sv	-
8540	E	96	0.532	68.9	211	5	28.5	III III	vv v	sr

Table 9. (continued) Vibration Study, Variable Inductance Pickup Method,

Spindle Bearing Friction Curve, Tabulated Data

Speed	Oil	Temp.	Cord Load	Sommerfeld Number	Chart Reading	Scale Length	Coef. of Friction Variable	Oscillograph Observations			Chart[c] Remarks
								Type[a]	Remarks[b]		
rpm		°F	lbs.	sec/min							
9440	E	99	0.532	73.5	222	5	30.0	III III	sv -		-
10360	E	100	0.532	79.2	285	5	38.2	III III	- -		sr
11250	E	104	0.532	78.2	241	5	32.2	III III	- -		sr
7610	E	97	3.035	10.88	189	5	4.44	II III	sv -		-
8570	E	98	3.035	12.09	189	5	4.44	II III	sv -		-
9500	E	100	3.035	12.78	210	5	5.97	III III	sv -		sr
10460	E	99	3.035	14.30	225	5	6.41	III III	- -	M	sr
11390	E	104	3.035	13.90	215	5	6.08	III III	- -	N	sr
9490	E	111	5.031	6.15	340	5	4.79	V VI	vv v		su
10460	E	105	5.031	8.34	270	5	3.84	I III	v -		sr
11390	E	110	5.031	7.00	290	5	4.13	II VI	- -	P	sr
11390	E	124	7.029	4.25	360	5	3.67	V III	v -	R	vu

Remarks:

[a]Type of wave form:

Type I 

Type II 

Type III 

Type IV 

Type V 

Type VI 

Upper line of a run is with the pickup in position A; lower line, position B.

[b]Mark	Meaning
v	trace vibrates or there are several parallel traces or both.
c	trace tends to change types (see above).
o	alternate peaks move slowly up and down out of phase with each other (indicates oil-film whirl).

Prefixes same as those in footnote [b], Table 8.

Capital letters refer to Table 10.

[c]See footnote [b], Table 8.

Table 10. Vibration Study, Runs with Oil Whip, Additional Data

Mark	Scale Length of Running Speed Vibration	Number of Cycles per L Units Scale Length		Frequency cyc/sec
A	18	6 in 8.5		4260
	18	8 in 11		4390
B	19	20 in 34		3560
	19	21 in 35.67		3660
C	18.75	8 in 13		3570
	18.75	23 in 34.5		3870
D	18.75	9 in 15		3790
	18.75	7 in 12		3700
E	20.5	4 in 4	?	4800
	20.5	5 in 4	?	6000
F	21	---------		----
	21	14 in 24		3000
G	20.5	3 in 5.75		2980
	20.5	3 in 4		4290
H	20	1 in 1	?	4460
	20	7 in 7	?	4460
J	20.5	--------		----
	20.5	4 in 5	?	3860
K	20	5 in 8		3400
	19.5	7 in 11.5		3390
L	20	16 in 29		3200
	20.5	20 in 38		3050
M	20	2 in 3.5	?	3080
	20.5	---------		----
N	----	---------		----
	20.5	2 in 2.5	?	5660
P[a]	20.5	4 in 8	?	2900
	20	20 in 37		3110
R	20.75	---------		----
	20.75	---------		----

[a]The vibration stopped and the torque dropped markedly before the chart was started.

Table 11. Vibration Study, Runs with Oil Whip

Effect of Speed and Oil

	Oil	Approximate Speed			
		8500	9500	10,500	11,500
Frequencies	A			4260	3560
				4390	3660
				3570	3790
				3870	3700
			4800 ?	3000	2980
					4290
Average				3813	3663
			(4800)		
	H	4460 ?	3860 ?	3400	3200
		4460 ?		3390	3050
Average				3395	3125
		(4460)	(3860)		
	E			3080 ?	
					2900?
					3110
Average					3110
				(3080)	(3005)

The upper average is the average with the questioned frequencies omitted. The lower average is the average with only those values which are obviously in error (those values crossed out) omitted.

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